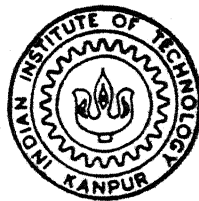


# PASSIVE ATTITUDE CONTROL THROUGH TETHER FOR SATELLITES IN ELLIPTIC ORBITS

*by*

**KAILASH CHAUDHARY**



**DEPARTMENT OF AEROSPACE ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY KANPUR**

**DECEMBER, 1993**

AE

1993

M

CHA

PAS

THI  
AE/1993/m  
C393P

PASSIVE ATTITUDE CONTROL THROUGH TETHER  
FOR SATELLITES IN ELLIPTIC ORBITS

*A Thesis Submitted*  
in Partial Fulfilment of the Requirements  
for the Degree of  
MASTER OF TECHNOLOGY

00000

by

KAILASH CHAUDHARY

to the

DEPARTMENT OF AEROSPACE ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

December 1993

TH  
629.411  
C393p

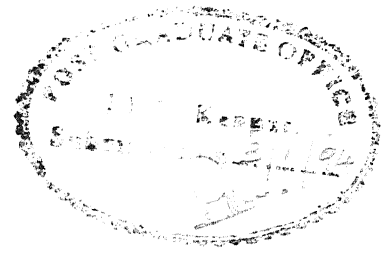
- 4 MAR 1994

CENTRAL LIBRARY  
U. T. KANPUR

Acc. No. **A. 117459**

AE-1993-m-CNA-PASS

CERTIFICATE



This is to certify that the work **Passive Attitude Control Through Tether for Satellites in Elliptic Orbits** , has been carried out under my supervision and has not been submitted elsewhere for award of a degree.

*Krishna Kumar*

( Krishna Kumar )

Professor

Dept. of Aerospace Engg.

I. I. T. Kanpur

India

December. 1993

## ABSTRACT

Satellite pointing precision that can be achieved through gravity gradient moments is rather limited, especially in presence of disturbances. The gravity gradient stabilization in elliptic orbits beyond eccentricities of 0.30 is not possible. Besides, an effective utilization of this attitude control concept demands slender satellite shapes thus imposing a major design constraints. Here, we propose a new approach for satellite attitude control that overcomes most of these limitations. This concept utilizes a small mass and a pair of tethers for satellite attitude stabilization. The configuration considered for the suggested application assumes a downward "pendulum like" deployment of the auxiliary mass from the satellite through the pair of identical tethers attached to its distinct and symmetrically offset points around the mass centre.

In order to examine the applicability of the proposed passive attitude control concept, to elliptic orbits, the case of in-plane pitching librations is considered. The Lagrangian formulation is adopted to obtain the system equations of motion. The governing set of non-linear, non-autonomous set of equations is solved numerically. The simulations establish the continuing effectiveness of the proposed concept for elliptic orbits and ensure a high degree of pointing stability.

*Dedicated to ....*

My Parents .

## ACKNOWLEDGEMENTS

I express my deep gratitude to Prof. Krishna Kumar, my thesis supervisor, for his valuable guidance and constant encouragement through out this work. I have the highest regards for his openness to discussion, patience towards my riders, granting freedom to work independently and constructive criticism from time to time. Working with him has indeed been an enriching experience for me.

I gratefully acknowledge the help rendered by Mr. S. P. Chandra Sekhar, Mr. Hemant Kumar, Mr. K. D. Kumar and Mr. Ravindra Prasad towards preparation of this document. I express my thanks to all my MITian friends, Mr. A. K. Choudhary, Mr. S. K. Singh, Mr. Vikash Kumar, Mr. Himanshu Shekhar and Mr. Amit Sahay who have been supportive throughout and shared my joys and sorrows during this period. My other friends whose name I could not include provided me memories to cherish, a long time to come.

Last but not the least, my elder brother deserves the deepest appreciation for his love and guidance.

December, 1993

Kailash Chaudhary

## CONTENTS

CHAPTER 1	1-4
INTRODUCTION	
1.1 Preliminary Remarks	1
1.2 Some Important TSS Applications	1
1.3 Brief Review of the Literature	3
1.4 Purpose and Scope of Investigation	4
CHAPTER 2	5-17
DYNAMICS OF TWO BODY BI-TETHERED SYSTEM	
2.1 Introduction	5
2.2 Description of the System	5
2.3 Formulation	6
2.3.1 Potential Energy of the System	7
2.3.2 Kinetic Energy of the System	9
2.4 The Lagrangian Equations of Motion	10
2.5 Numerical Integration	12
2.6 Results and Discussion	13
CHAPTER 3	18-21
TETHER DEPLOYMENT AND RETRIEVAL ANALYSIS	
3.1 Introduction	18
3.2 Modified Constraints	18
3.3 Results and Discussion	19
3.3.1 Tether Deployment Case	19
3.3.2 Tether Retrieval Case	20



CHAPTER 4	22-23
CONCLUDING REMARKS	
Scope for Future Work	23
REFERENCES	24-26

## LIST OF FIGURES

Figure No.	Page
1. Geometry of Satellite Carrying bi-tethered auxiliary mass in an elliptic orbit	27
2. Typical $\alpha$ , $\beta$ pitch angle response for Satellite in elliptic orbit	28
3. Typical Satellite librational response for various tether lengths	29
4. Satellite librational response for various tether lengths at eccentricity, $e=0.05$	30
5. Satellite librational response for various tether lengths at eccentricity, $e=0.10$	31
6. Librational response for Satellites in orbits of various eccentricities at $L_{jo}=10$ km	32
7. Librational response for Satellites in orbits of various eccentricities at $L_{jo}=1$ km	33
8. Typical librational response for various Satellite mass distributions at $e=0.1$	34
9. Librational response for various Satellite mass distributions at $e=0.05$	35
10. Satellite librational response for various Tether offsets	36

11. Satellite librational response for various impulsive disturbances at $L_{jo} = 10$ km	37
12. Satellite librational response for various impulsive disturbances at $L_{jo} = 0.5$ km	38
13. Librational response $\alpha$ for different configurations and disturbances under deployment	39
14. Librational response $\alpha$ for different configurations and mass distributions under deployment	40
15. Librational response $\beta$ for different configurations and disturbances under deployment	41
16. Librational response $\beta$ for different configurations and mass distributions under deployment	42
17. Librational response $\alpha$ for different disturbances under retrieval	43
18. Librational response $\alpha$ for various impulsive disturbances and mass distributions under retrieval	44
19. Librational response $\beta$ for different disturbances under retrieval	45
20. Librational response $\beta$ for various impulsive disturbances and mass distributions under retrieval	46

## NOMENCLATURE

$a$	: Semi major axis of the elliptic orbit
$a_j$	: $j$ th Tether offsets
$\hat{a}_j$	: $a_j / L_{ref}$
C.M.	: Centre of mass of the system
$d_j$	: Diameter of $j$ th tether
$E_j$	: Young's modulus of elasticity
$e$	: Eccentricity of the orbit
$h$	: Specific angular momentum
$\hat{i}, \hat{j}, \hat{k}$	: Unit vector along coordinate axes $x, y, z$
$\hat{I}, \hat{J}, \hat{K}$	: Unit vector in the inertial frame of reference, O-XYZ
$I_{j,j}$	: $j$ th component of moment of inertia of the shuttle about axis passing through its C.M., $j=x, y, z$
$I_e$	: Mass distribution parameter
$K$	: Mass factor
$k$	: Rate of deployment/retrieval
$L_{j0}$	: Unstretched length of the $j$ th tether
$L_j$	: Stretched length of the $j$ th tether
$L_{ref}$	: Reference length
$l_j$	: $L_j / L_{ref}$
$M$	: Mass of the Main Satellite body
$m$	: Mass of the subsatellite
$\vec{R}$	: Position vector of the C.M. of the System
$T$	: Kinetic Energy of the System

$U$  : Potential Energy of the System

$U(\epsilon_j)$  : Unit function

$x,y,z$  : Body fixed coordinate system

$\theta$  : True anomaly measured from reference line

$\mu$  : Earth's gravitational constant

$\beta$  : Pitch response of the subsatellite

$\epsilon_j$  : Strain in jth Tether

$\dot{(\ )}$  :  $\frac{d}{dt} (\ )$

$\frac{d}{d\theta} (\ )$

# CHAPTER 1

## INTRODUCTION

### 1.1 Preliminary remarks :

We have come a long way in space research. Numerous technological applications influencing everyday life are already bringing about major changes in society. These developments often pose newer challenges. One such challenge lies in the use of tethers in satellite applications.

There are several Tethered Satellite System (TSS) missions which have been proposed and undertaken. The mission TSS-1 undertaken last year failed due to some obstruction coming in the way of the planned tether deployment. But, subsequently the feasibility of TSS has been established through deployment of spent rocket stage from a launch vehicle. The TSS-2 and TSS-3 are the next two important tethered satellite missions jointly planned by Italy and U.S.A..

### 1.2 Some Important TSS Applications (Ref-1)

Numerous applications of TSS have been proposed and analyzed. Some of the important ones are listed below:

(i) **Science applications:** It is possible to use the remote platforms deployed through long tethers for numerous scientific applications. It facilitates gathering valuable scientific data as

the universe through on-board instruments. In particular such systems would enable long term observations of various phenomena in the lower atmosphere.

(ii) **Transportation applications:** A tether can be used for boosting an orbiting satellite payload into a higher circular or elliptical orbit with a net saving of propulsive energy.

(iii) **Space propulsion applications :** The principle of electromagnetic interaction of a conducting tether with the interplanetary magnetic field can be used to generate propulsive forces for interplanetary travel. Of course, this would require the on-board electric power.

(iv) **Electrodynamics applications:** A long insulating tether revolving at high satellite speeds, cutting earth's magnetic lines of force induces an e.m.f., which in turn can be used as a source of electric power on-board. This however results in an electromagnetic drag causing the orbit to shrink. Thus orbital energy supplies the electric power.

(v) **Controlled gravity applications :** In such applications tethers are used for generating artificial gravity at various levels, especially the micro-gravity. This is achieved by a tethered platform, composed of two end structures which are connected by a deployable or retractable tether. The tether length is used to control the gravity level.

(vi) **Constellation applications:** It is possible to use tether for

deploying satellites in various constellations for centralized in-space services.

### 1.3 Brief Review of The Literature :

Von Tiesenhausen<sup>2</sup> and Baker<sup>3</sup> emphasized the use of tethers in space and hence the study of the tethered satellite systems. Colombo, et al<sup>4</sup> have shown that tethered satellite systems lead to a significant impulse saving for the transfer missions. Their two dimensional simulation assumes a massless tether, neglects tether dynamics, and integrates only the motion of two end masses. Mackinney and Tschirgi<sup>5</sup> while considering shuttle and space station developed tethered subsatellite systems-have also accounted for the tether mass along with gravity-gradient in their analysis. The attitude control applications of tethers<sup>6-14</sup> proposed are mostly based on the use of actuating feedback mechanisms. The system enables variation of the net moment acting satellite through tether tensions using a suitable feedback strategy as proposed by A. K. Misra ,V. J. Modi<sup>6</sup> and C. C. Rupp<sup>7</sup> . Kumar and Kumar<sup>8</sup> proposed the use of short tethers and small mass as an entirely passive satellite attitude control device in circular orbits. Kumar ,Jha ,Misra ,and Yan<sup>15</sup> analyzed the use of three body tethered system in conjunction with the space station .The paper aims at determining the frequencies of transverse oscillations of three body two-tethered system. C. Kowalsky and J. David Powell<sup>16</sup> laid emphasis on the design of the artificial



gravity spacecraft. Recently J. D. Osten and T. R. Kane<sup>17</sup> focussed on the refinements in the control strategy necessary to maintain a constant rotation rate of the system by means of tangential thrusters while simultaneously varying the length of the tether.

#### 1.4 Purpose and Scope of Investigation:

The present investigation attempts to extend the satellite attitude control application of tether to elliptic orbit. First, the pitching responses of two body two tethered system have been studied. The effects of various system parameters like length of tethers, mass distribution, tether offsets ,etc. on satellite attitude stabilization have been examined. The pitching response during the proposed deployment/ retrieval phases of the TSS has been studied in the next phase.

## CHAPTER 2

### DYNAMICS OF TWO BODY BI-TETHERED SYSTEM

#### 2.1 Introduction :

Dynamics of tethered system is rather complicated. It involves translational, angular and vibrational motions. The transverse vibrations increase the degree of complexity considerably. In the present analysis we ignore the lateral mode of oscillations of the tether. Care has been taken to take into account the energy associated with longitudinal tether oscillations as well as that for the satellite and auxiliary mass. For simplicity, the effects of oblateness of the earth, its magnetic field, gravitational pull of the other celestial bodies, solar radiation, and atmospheric drag have also been neglected in the present analysis.

#### 2.2 Description of The System :

The configuration of the system proposed consists of the main satellite body, an auxiliary body (subsatellite) and two identical elastic tethers. This general system considered revolves around the earth in general elliptic orbit. Only the in-plane pitching oscillations are investigated.

Several assumptions are made to facilitate the investigation. The upper main satellite body (  $M$  ) is assumed to be much heavier

than the subsatellite which is treated as a point mass. The tethers are taken to be massless and have linear elastic characteristics. The angle between the position vectors of the C.M. of the main satellite body and that of the whole system is treated as negligible. As usual the Reference Line for measuring the true anomaly is chosen along the perigee axis. The body fixed axes (frame) o-xyz are taken to be along the principal directions. The auxiliary mass (m) is deployed from the main satellite body through a pair of identical tethers, the other ends of which are attached to two symmetrical points on the z-axis on either side of its mass centre.

The co-ordinate system O-XYZ is an inertial frame with its origin fixed at the Earth's centre. The position of the C.M. of the system is denoted by the vector  $\vec{R}$ . The angle  $\alpha$  denotes the pitch angle of the main satellite body while the angle  $\beta$  denotes the pitch angle of the subsatellite (Fig. 1).

### 2.3 Formulation:

The Lagrangian approach has been used to develop the equations of motion. Let us first look at the geometrical constraints<sup>8</sup> of the system which can be written as :

$$L_1 = L_{10} (1 + \varepsilon_1) = [a_1^2 + L^2 - 2 a_1 L \sin(\alpha - \beta)]^{1/2}$$

$$L_2 = L_{20} (1 + \varepsilon_2) = [a_2^2 + L^2 + 2 a_2 L \sin(\alpha - \beta)]^{1/2}$$

where.

$a_1, -a_2$  = Z co-ordinates of the attachment points

$\epsilon_1, \epsilon_2$  = strain in the first and second tether

$L_{10}, L_{20}$  = Unstretched tether lengths

$L_1, L_2$  = Stretched tether lengths

### 2.3.1 Potential Energy of The System:

The total Potential energy ( PE ),  $U$  of the system is the sum of the gravitational PE of the main satellite (  $U_M$  ) subsatellite (  $U_m$  ) and elastic strain energy of the two tethers (  $U_s$  ). In other words

$$U = U_M + U_m + U_s$$

where,

$$U_M = - \int \frac{\mu}{|\vec{r}_1|} dM$$

$$U_m = - \int \frac{\mu}{|\vec{r}_2|} dm$$

$$U_s = \frac{1}{2} \sum_j E_j A_j \epsilon_j^2 L_{j0} U(\epsilon_j)$$

where  $j = 1, 2$

$$U(\epsilon_j) = 0, \text{ if } \epsilon_j = 0$$

$$= 1, \text{ if } \epsilon_j > 0$$

$\vec{r}_1$  = position vector of an elemental mass of the main satellite

$$\begin{aligned}
&= \vec{r}_s + \vec{r} \\
&= \vec{R} - K \vec{L} + \vec{r} \\
&= R \cos \alpha \hat{j} - R \sin \alpha \hat{k} - K L \{ -\cos(\alpha - \beta) \hat{j} + \sin(\alpha - \beta) \hat{k} \} \\
&\quad + x \hat{i} + y \hat{j} + z \hat{k}
\end{aligned}$$

where,

$$K = \frac{m}{M + m} \text{, mass factor, showing the shifting of C.M. of}$$

the system from the main satellite body.

$\vec{R}$  = position vector of the C. M. of the system.

$\vec{L}$  = position vector of the auxiliary mass from the  
C.M. of the main satellite.

$\vec{r}_2$  = position vector of the subsatellite (m)

$$= \vec{r}_s + \vec{L}$$

$$= \vec{R} + (1-K) \vec{L}$$

$$= R \cos \alpha \hat{j} - R \sin \alpha \hat{k} - (1-K) L \cos(\alpha - \beta) \hat{j} + (1-K) \sin(\alpha - \beta) \hat{k}$$

On carrying out necessary algebraic calculations and simplifications, we get

$$\begin{aligned}
U = & \left[ -\frac{\mu (M + m)}{R} + \mu m L^2 \{ (1-K)^2 - 3 (1-K) \cos^2 \beta \} / 2 R^3 \right. \\
& + \mu (-2 I_{xx} + I_{yy} + I_{zz}) / 4 R^3 \\
& + 3 \mu (I_{yy} - I_{zz}) \cos 2\alpha / 4 R^3 \\
& - 3 \mu K L \cos \beta (I_{xx} + I_{yy} + I_{zz}) / 4 R^4 \\
& + \mu M K^2 L^2 \left( 1 - \frac{3 K L \cos \beta}{R} \right) / 2 R^3 \\
& \left. + \frac{1}{2} \left\{ \sum_{j=1}^2 E_j A_j L_j \varepsilon_j^2 U(\varepsilon_j) \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& - 3 \mu K L \{ I_{xx} \cos \beta - (I_{yy} - I_{zz}) \cos(2\alpha - \beta) \} / 2 R^4 \\
& - 3 \mu K L^2 \{ I_{xx} - (I_{yy} - I_{zz}) \cos 2(\alpha - \beta) \} / 4 R^5
\end{aligned}
\quad ]$$

### 2.3.2 Kinetic Energy of The System:

The total Kinetic Energy ( KE ) of the system is the sum of the KE of the main satellite ( M ) and subsatellite (m) represented by  $T_M$  and  $T_m$  respectively. In other words

$$T = T_M + T_m$$

where ,

$$T_M = \frac{1}{2} \int \left| \dot{\mathbf{r}}_1 \right|^2 dM$$

$$T_m = \frac{1}{2} \int \left| \dot{\mathbf{r}}_2 \right|^2 dm$$

The total KE of the system can also be written in another way, however it gives the same result. The total KE of the system is the sum of translational KE and rotational KE of both the main satellite and subsatellite. For convenience, either inertial or non-inertial frame of reference can be used.

Thus (in inertial frame O-XYZ)

$$T = \frac{1}{2} M \left| \dot{\mathbf{r}}_s \right|^2 + \frac{1}{2} I_{xx} \left| \dot{\omega} \right|^2 + \frac{1}{2} m \left| \dot{\mathbf{r}}_z \right|^2$$

where,

$$\begin{aligned}
 \vec{r}_s &= \vec{R} - K \vec{L} \\
 &= R \cos\theta \hat{J} + R \sin\theta \hat{K} - K \{ -L \cos(\theta + \beta) \hat{J} - L \sin(\theta + \beta) \hat{K} \} \\
 \dot{\vec{r}}_s &= [ \dot{R} \cos\theta - R \dot{\theta} \sin\theta + K \dot{L} \cos(\theta + \beta) - KL (\dot{\theta} + \dot{\beta}) \sin(\theta + \beta) ] \hat{J} \\
 &\quad + [ \dot{R} \sin\theta + R \dot{\theta} \cos\theta + K \dot{L} \sin(\theta + \beta) + KL (\dot{\theta} + \dot{\beta}) \cos(\theta + \beta) ] \hat{K} \\
 \vec{r}_z &= \vec{R} + (1-K) \vec{L} \\
 &= [ R \cos\theta - (1-K) L \cos(\theta + \beta) ] \hat{J} + \\
 &\quad [ R \sin\theta - (1-K) L \sin(\theta + \beta) ] \hat{K} \\
 \dot{\vec{r}}_z &= [ \dot{R} \cos\theta - R \dot{\theta} \sin\theta - (1-K) \{ \dot{L} \cos(\theta + \beta) \\
 &\quad - L (\dot{\theta} + \dot{\beta}) \sin(\theta + \beta) \} ] \hat{J} \\
 &\quad + [ \dot{R} \sin\theta + R \dot{\theta} \cos\theta - (1-K) \{ \dot{L} \sin(\theta + \beta) \\
 &\quad + L (\dot{\theta} + \dot{\beta}) \cos(\theta + \beta) \} ] \hat{K} \\
 \dot{\omega} &= (\dot{\theta} + \dot{\alpha}) \hat{I}
 \end{aligned}$$

On carrying out algebraic calculations and necessary simplifications, we get

$$\begin{aligned}
 T = \frac{1}{2} (M + m) ( \dot{R}^2 + R^2 \dot{\theta}^2 ) + \frac{1}{2} (1-K) m \{ \dot{L}^2 + L^2 (\dot{\theta} + \dot{\beta})^2 \} \\
 + \frac{1}{2} I_{xx} (\dot{\theta} + \dot{\alpha})^2
 \end{aligned}$$

#### 2.4 The Lagrangian Equations of Motion:

The equations of motion can be obtained from

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j$$

where,

$L$  = Lagrangian of the system,  $(T - U)$

$$= L(\dot{q}_j, q_j)$$

$q_j$  = generalized coordinate (independent variable)

$Q_j$  = The generalized force correspond to  $j$ th generalized coordinate.

Here, we have mass factor  $K \ll 1$ , since the main satellite body is assumed to be much heavier than the subsatellite. Hence  $K$  is put equal to zero into the expression for  $L$  before putting it into the Lagrangian equation for simplicity.

The resulting equations of motion for  $\alpha$ ,  $\beta$  and  $L$  are :

$$\ddot{\alpha} + \ddot{\theta} - 3(\mu/R^3) I_y \sin\alpha \cos\alpha + [(\Lambda_z a_z - \Lambda_1 a_1)(L/I_{xx}) \cos(\alpha - \beta)] = 0$$

$$\ddot{\beta} + \ddot{\theta} + 3(\mu/R^3) \sin\beta \cos\beta + 2(\dot{\theta} + \dot{\beta})(\dot{L}/L) - [(\Lambda_z a_z - \Lambda_1 a_1)(1/mL) \cos(\alpha - \beta)] = 0$$

$$\ddot{L} + [(\Lambda_1 + \Lambda_2)/m - (\dot{\theta} + \dot{\beta})^2 + (1 - 3\cos^2\beta)(\mu/R^3)] L + (\Lambda_z a_z - \Lambda_1 a_1)(1/m) \sin(\alpha - \beta) = 0$$

$$\Lambda_j = E_j A_j \epsilon_j U(\epsilon_j) / L_j ; j = 1, 2$$

The above equations of motion can be written in non-dimensionalized form using following relations :

$$R^2 \dot{\theta} = h = \text{constant}$$

$$l/R = 1 + e \cos\theta$$

$$l = a(1 - e^2) = h^2/\mu$$



where,

$l$  = semi letus rectum

$\mu$  = earth's gravitational constant

$e$  = eccentricity of the orbit

The equations in the non dimensional form can be written as :

$$(1 + e \cos \theta) \alpha'' - 2 e (1 + \alpha') \sin \theta - 3 \frac{I_r}{r} \sin \alpha \cos \alpha \\ = (\lambda_1 - \lambda_2) l \cos(\alpha - \beta) / (1 + e \cos \theta)^3$$

$$(1 + e \cos \theta) \beta'' + \beta' [2 (1 + e \cos \theta) l' / l - 2 e \sin \theta] \\ + 3 \sin \beta \cos \beta + 2 (1 + e \cos \theta) l' / l \\ = 2 e \sin \theta - (\lambda_1 - \lambda_2) \cos(\alpha - \beta) / \{l(1 + e \cos \theta)^3\}$$

$$(1 + e \cos \theta) l'' - 2 e \sin \theta l' - (1 + e \cos \theta) (1 + \beta')^2 l \\ + (1 - 3 \cos^2 \beta) l = - [ \lambda_1 \{1/\hat{a}_1 - \sin(\alpha - \beta)\} + \\ \lambda_2 \{1/\hat{a}_2 + \sin(\alpha - \beta)\} ] \{1/(1 + e \cos \theta)^3\}$$

where,

$$\lambda_j = \frac{C_j \varepsilon_j U(\varepsilon_j)}{l_j} ;$$

$$C_j = \frac{E_j A_j a_j^3 (1 - e^2)^3}{I_{xx} \mu} ; \dots j = 1, 2$$

## 2.5 Numerical Integration:

The three variables in the formulation are generalized co-ordinates i.e.  $\alpha$ ,  $\beta$  and  $l$ . The angle  $\theta$  is the independent

variable. The other variables like  $l_1, l_2, \epsilon_1, \epsilon_2$  in the above three equations are dependent variables and can be evaluated from the constraint equations. The solution of the governing set of equations with twin tether length constraints is attempted using Runge-kutta Fourth order Method of solving simultaneous Ordinary differential equations. During the integration the zero initial values are assumed for the longitudinal tether strains and their derivatives.

## 2.6 Results and Discussion :

The numerical integration undertaken, covers a wide range of important system parameters; e.g. eccentricities, tether lengths, offsets, satellite mass distribution, etc . Some important typical results are presented for the system described below :

**Satellite :** The space shuttle representing the main satellite with an auxiliary mass attached through a pair of identical tethers is taken as a particular case.

We assume

$$I_{xx} = I_{yy} = 10^7 \text{ kg m}^2, I_{zz} = 6 \times 10^5 \text{ kg m}^2$$

$$\text{Offsets } a_1 = a_2 = 10 \text{ m}$$

**Orbit :** Semi major axis = 6620 km

**Tether:** Material : stainless steel

$$\text{Young's modulus of elasticity; } E_1 = E_2 = 2 \times 10^{11} \text{ N/m}^2$$

$$\text{diameter} = 0.5 \text{ mm.}$$

**Aux. mass:** 150 kg

The assumed satellite mass distribution leads to

$$I_r = (I_{yy} - I_{zz}) / I_{xx} = +0.94.$$

When the axis of minimum moment of inertia is oriented along the z-axis taken along the local horizontal in the desired satellite orientation, it leads to a positive inertia parameter  $I_r$ . However, this corresponds to a highly unstable equilibrium configuration in the gravity gradient sense. On the other hand, when this long axis is aligned with the y-axis taken along the local vertical, the resulting satellite orientation is stable and is associated with a negative  $I_r$  value.

We now begin with numerical simulation for the case of the slender satellite in an elliptic orbit with the long satellite axis positioned along the local horizontal with  $I_r = 0.94$ . Besides considering such a highly unstable orientation, we assume a rather large initial disturbance. The tether offsets of 10 m are taken along with a modest tether length of 5 km. It is interesting to note that the pitching angles  $\alpha$  and  $\beta$  exhibit similar stable oscillating behaviour with rather low amplitudes (Fig. 2).

Next, we examine the influence of varying tether lengths on satellite attitude motion (Fig. 3-5). An excessive increase in the tether lengths to say 100 km causes the beneficial stabilizing influence on pitching motion due to the tether tensions to diminish significantly and slowly growing amplitudes are observed. This may be attributed to the highly reduced level of differential between

the strains in the two tethers and hence significantly reduced restoring moments. An excessive decrease in the tether lengths to say 100 m again results in satellite instability. This may be explained by tensions and hence the stabilizing moment levels becoming too low. These results that the proposed control mechanism would in general be most effective when moderate tether lengths are taken. A tether length ranging from say 500 m to 10 km appears to be appropriate for circular orbits i.e. eccentricity zero. However, for a higher elliptic orbit say  $e = 0.05$ , the tether length of 1 km to 10 km seems to be appropriate, (Fig. 4). But, for eccentricity  $(e)=0.1$  the tether length of about 10 km is appropriate, (Fig. 5). Thus, in general, it can be inferred that the more the satellite departures from circular orbits the larger is the length of tethers required. This may be attributed to the disturbance caused due to variable angular speed ( $\dot{\theta}$ ) of the satellite system.

Fig.(6-7) present the stabilizing influence of tether tensions as affected by varying orbital eccentricities. It may be observed that the satellite continues to be stable even for the adverse mass distribution assumed for the satellite in highly elliptic orbits and large initial impulsive disturbance. In general, an increase in eccentricity leads to an increase in the librational amplitudes. When the eccentricity is too large, the proposed control fails to ensure satellite pointing stability. Fortunately, for practical applications normally involving satellites in the orbits of

modest eccentricities, this may not limit the usefulness of the concept.

The influence of varying satellite mass distribution on the control effectiveness of the proposed concept is studied in Fig. (8-9). It is interesting to note that the satellite attitude characteristics remain virtually independent of the satellite mass distributions considered. This result is of considerable significance. Evidently, the proposed stabilizing mechanism offers significant advantages over the pure gravity gradient system. The mass distribution constraint for satellite pointing stability is now virtually eliminated.

Fig. (10) shows the effect of the tether offsets on the pointing stability of satellites positioned along the normally unstable orientation corresponding to  $I_r = 0.94$ . Evidently, the choice of an offset above a certain minimum value leads to satellite pointing stability. On the other hand, in situations where the offsets are too small, the satellite may undergo large amplitude/tumbling motions. This occurs essentially due to the control moments generated through tether tension falling below the required level. Thus, sufficiently large tether offsets must be chosen in order to generate significant control moments that would restrict the pitch angle to low values.

Fig. (11-12) present a comparative assessment of satellite pitching performance in presence of large initial librational

disturbance for a tether length of 10 km and 0.5 km, the tether offset of 10 m and  $I_r = 0.94$ . It may be worthwhile to point out that the order of magnitude of the librational amplitude remains the same for all the cases with tether length of 10 km whereas for tether length of 0.5 km the pitching responses are unstable. Furthermore the larger disturbances do not necessarily lead to a larger amplitude motion for the former one. This is somewhat unexpected and perhaps attributed to the strongly non-autonomous character of the dynamic system.

## CHAPTER 3

### TETHER DEPLOYMENT AND RETRIEVAL ANALYSIS

#### 3.1 Introduction :

In the previous chapter the dynamics of two body bi-tethered system for a deployed tether case has been studied. In this case the nominal tether lengths are not varying with true anomaly ( $\theta$ ). In this section, we will focus our attention on the pitching response of the system while simultaneously varying these lengths of the tethers. The exponential law has been assumed for the tether deployment/ retrieval.

#### 3.2 Modified constraints :

During deployment or retrieval of the tethers, two more equations are added which in non-dimensional form can be written as:

$$l_j(\theta) = l_{j0} \exp(k\theta) \quad ; j = 1, 2$$

Now, in addition to the set of three equations of motion for  $\alpha$ ,  $\beta$  and  $l$ , the constraint equations can be written as :

$$l_1(\theta) = L_{10} \exp(k\theta) (1 + \epsilon_1) = \left[ \hat{a}_1^2 + l^2 - 2 \hat{a}_1 l \sin(\alpha - \beta) \right]^{1/2}$$

$$l_2(\theta) = L_{20} \exp(k\theta) (1 + \epsilon_2) = \left[ \hat{a}_2^2 + l^2 + 2 \hat{a}_2 l \sin(\alpha - \beta) \right]^{1/2}$$

where,

$l_{j0}$  = initial non-dimensional length of the  $j$ th tether.

$k$  = ratio of deployment/retrieval rate and the tether length.

The parameter  $k$  chosen here is positive for deployment and negative for retrieval. For the deployed tether case considered in the last chapter,  $k$  becomes zero.

### 3.3 Results and Discussion :

#### 3.3.1 Tether Deployment Case :

During deployment of the proposed two tether mass system, the length of the tethers are increased exponentially with independent variable  $\Theta$ . The system with the shuttle long axis aligned with the local horizontal ( $I_r = 0.94$ ), the satellite pitch angle tends to grow and attains unacceptably large values, (Fig. 13). However, the pitch angle response is within stable range for the initial configuration i.e. the angle  $\alpha_0 = 0$ . The above result is in direct contrast with the well known stable behaviour of the simple tether case. This may be due to two factors. First, the shuttle configuration being considered is an unstable one. Second, slackening in the tethers during deployment causes the tensions and hence the stabilizing moment to disappear.

Next, we consider the case when the shuttle is aligned with the local vertical ( $I_r = -0.94$ ) although the tether attachment points still remaining on the local horizontal axis. (Fig. 14). Even in th



absence of the initial disturbance, the shuttle shows a stable pitching response for the orientation of  $\alpha_0 = 0^\circ$ . However, for the configuration of  $\alpha_0 = 90^\circ$ , the response is unstable. The earlier stable result corresponding to  $\alpha_0 = 0^\circ$  may be attributed to the favourable gravity gradient moments.

In all the above cases the pitching response  $\beta$  of the subsatellite remains virtually independent of the initial disturbances and orientation of main satellite body, (Fig.15-16). This amplitude of pitching angle of subsatellite for  $\alpha = 90^\circ$  is unexpectedly higher. This may be accompanied by exceptionally high rate of growth of  $\beta$  implying rotating tethers.

### 3.3.2 Tether Retrieval Case :

For the shuttle aligned with the local horizontal direction i.e.  $I_r = 0.94$ , retrieval of tethers is stable operation although the amplitude of  $\alpha$  is objectionably high, (Fig. 17-18). Here, it may be worth mentioning that the retrieval of tethers in a single tether two body case when tether is connected to the C.M. of the upper body, is an unstable operation. Further, in the present case as mentioned earlier, the retrieval of tethers would produce high tension in tethers. That partly explains the shuttle responses as observed here.

The pitching responses seem to be independent of initial disturbances. This may be possible due to high disturbance produced

by higher rate of retrieval of tethers. However, during retrieval with shuttle initially positioned in the nominally stable vertical configuration irrespective of the attachment points lying along the horizontal/vertical axis, pitching stability is observed. Also, in this case the pitching responses of subsatellite body seem to be independent of initial disturbances and changes in the shuttle mass distribution parameter, (Fig. 19-20).

117459

## CHAPTER 4

### CONCLUDING REMARKS

Some of the important features of the analysis and the results based on these may be summarized as follows :

- a. The proposed control mechanism provides significant control torque for satellite attitude stabilization. The moments induced by self adjusting tether tensions ensure that the attitude control effectiveness is virtually independent of the satellite mass distribution. As a result much higher attitude accuracies are expected even in the presence of moderate disturbances and/or orbit of significant eccentricity.
- b. The large increase in the attitude control torques achieved through the proposed bi-tethered mass attachment substantially limits the satellite excursions in the face of excitation, even in significantly elliptic orbits.
- c. It may be desirable to provide maximum tether offsets possible and consistent with the overall satellite layout design. Of course, these values should be duly checked for stability and acceptability of librational amplitudes through simulations.
- d. The relatively small auxiliary mass and tether lengths needed

for the open-loop stabilization of satellites even in the strongly unstable equilibrium configuration make the proposed level concept particularly attractive.

e. The space and weight penalties associated with the tethered mass are expected to be rather modest. Sometimes, it may be possible to reduce these further by shifting a part of the useful satellite mass to play the role of the "pendulum".

#### Scope for Further Work

The out of plane motion is of high importance for station keeping phase. A study using a more sophisticated model which includes the transverse vibrations of tether, structural damping of the system and roll motion of the system is recommended. Besides this, attention should also be focussed upon the use of tethers for artificial gravity problems, suitable deployment schemes and other scientific applications.

## REFERENCES

1. "Tethers in Space Handbook" by NASA, Aug. 1986.
2. Von Tiesenhausen, G. "Future Application of Tethers in Space"  
AIAA 24th Aerospace Sciences Meeting, Reno , Nevada, Jan. 1986.  
Paper No. 86-0053.
3. Baker, W. P., Dunkin, J. A., Galaboff, Z. J., Johnson, K. D.,  
et al , "Tethered Subsatellite Study" , NASA , TMX - 73324,  
Marshall Space Flight Center, Alabama . March 1976.
4. Colombo , G., et al " Use of Tethers for Payload Orbit Transfer  
" . Smithsonian Astrophysical Observatory, Cambridge , Ma, NAS  
8-33691, March , 1982.
5. Mckinney, L. E. and Tschirgi, " Tethers Deployed SSUS-A (Spin  
Stabilized Upper Stage " , McDonnell Douglas Corp., Huntington  
Beach CA. NAS 8-32842, April, 1984.
6. Misra, A. K. and Modi, V. J., " A Survey on the Dynamics and  
Control of Tethered Satellite Systems " Advances in the  
Astronautical Sciences , Vol. 62, edited by P. M. Bainum, et al  
.. American Astronautical Society, 1987. pp. 667-719.
7. Rupp., C. C., " A Tether Control Law for Tethered Subsatellite  
Along Local Vertical ", NASA TMX - 64963, Marshall Space Flight  
Center, Alabama , Sept. 1975.
8. Kumar, K. and Kumar, R., " Tether as an Open Loop Satellite  
Attitude Stabilizer : A Novel Concept ", under consideration for

- publication in Journal of Guidance, Control and Dynamics.
9. Bainum, P. M. and Kumar, V. K., " **Optimal Control of the Shuttle-Tethered System** ", Acta Astronautica. Vol. 7, No. 12, 1980, pp. 1333-1348.
  10. Banerjee, A. K. and Kane, T. R., " **Tethered Satellite Retrieval with Thruster Augmented Control**", Journal of Guidance, Control and Dynamics, Vol 7, No. 1, 1984, pp. 45-50.
  11. Xu, D. M.; Misra, A. K. and Modi, V. J., " **Thruster Augmented Active Control of a Tethered Subsatellite System During its Retrieval** ", Journal of Guidance, Control and Dynamics, Vol 9, No. 6, 1986, pp. 663-672.
  12. Lakshmanan, P. K., Modi, V. J. and Misra, A. K., " **Dynamics and Control of the Tethered Satellite in the Presence of Offsets**", Paper No. AAS 87-434, presented in the AAS/ALAA Astrodynamics Specialist Conference, Kalispell, Montana, Aug. 1987.
  13. Ruying, F. and Bainum, P. M., " **Dynamics and Control of a Space Platform with a Tethered Subsatellite** ", Journal of Guidance, Control and Dynamics, Vol 11, No. 4, 1988, pp. 377-381.
  14. Misra, A. K., and Diamond, G. S., " **On the Dynamics of a Subsatellite System Supported by Two Tethers** ", Journal of Guidance, Control, and Dynamics, Vol 9, No. 1, 1986, pp. 12-16.
  15. Kumar, K., Jha, O. N., Misra, A. K. and Yan, Y., " **Transverse Elastic Oscillations of Three Body Tethered Systems** ", Astrodynamics 1991, Vol 76, Advances in Astronautical Sciences.

16. Kowalsky, C. and Powell, J. D., " Tethered Artificial Gravity  
Spacecraft Design ", AAS-701, AAS/AIAA Astrodynamics  
Specialist Conference, Victoria, B. C., Canada, Aug 16-19, 1993.
17. Stoen, J. D. and Kane, T. R., " Spin Augmented Deployment and  
Retrieval of Tethered Artificial Gravity Spacecraft ". AAS-733.  
AAS/AIAA Astrodynamics Specialist Conference . Victoria, B. C.,  
Canada, Aug 16-19, 1993.

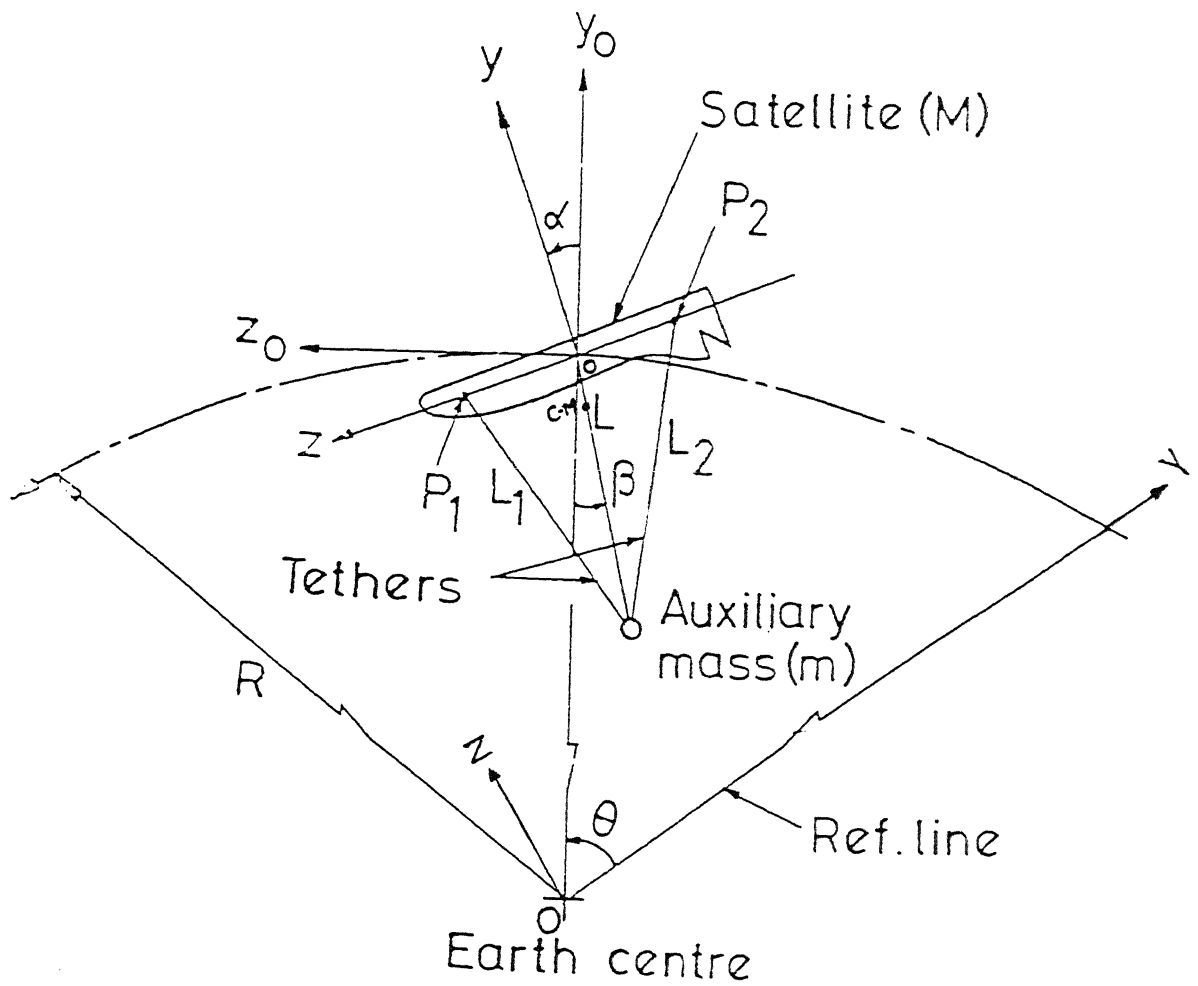


Fig.1. Geometry of Satellite Carrying bi-tethered auxiliary mass in an elliptic orbit



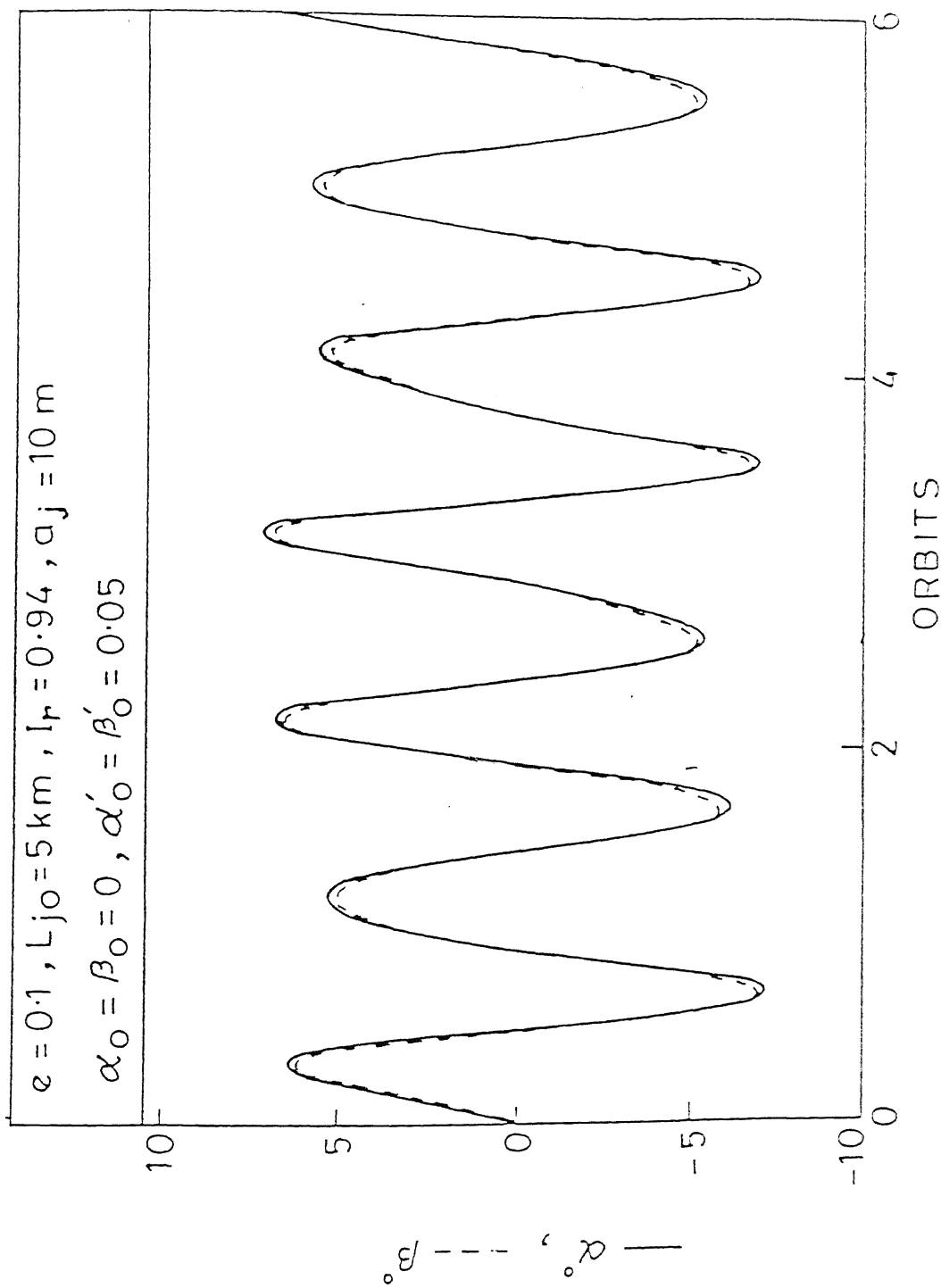


Fig.2. Typical  $\alpha, \beta$  pitch angle response for Satellite in elliptic orbit

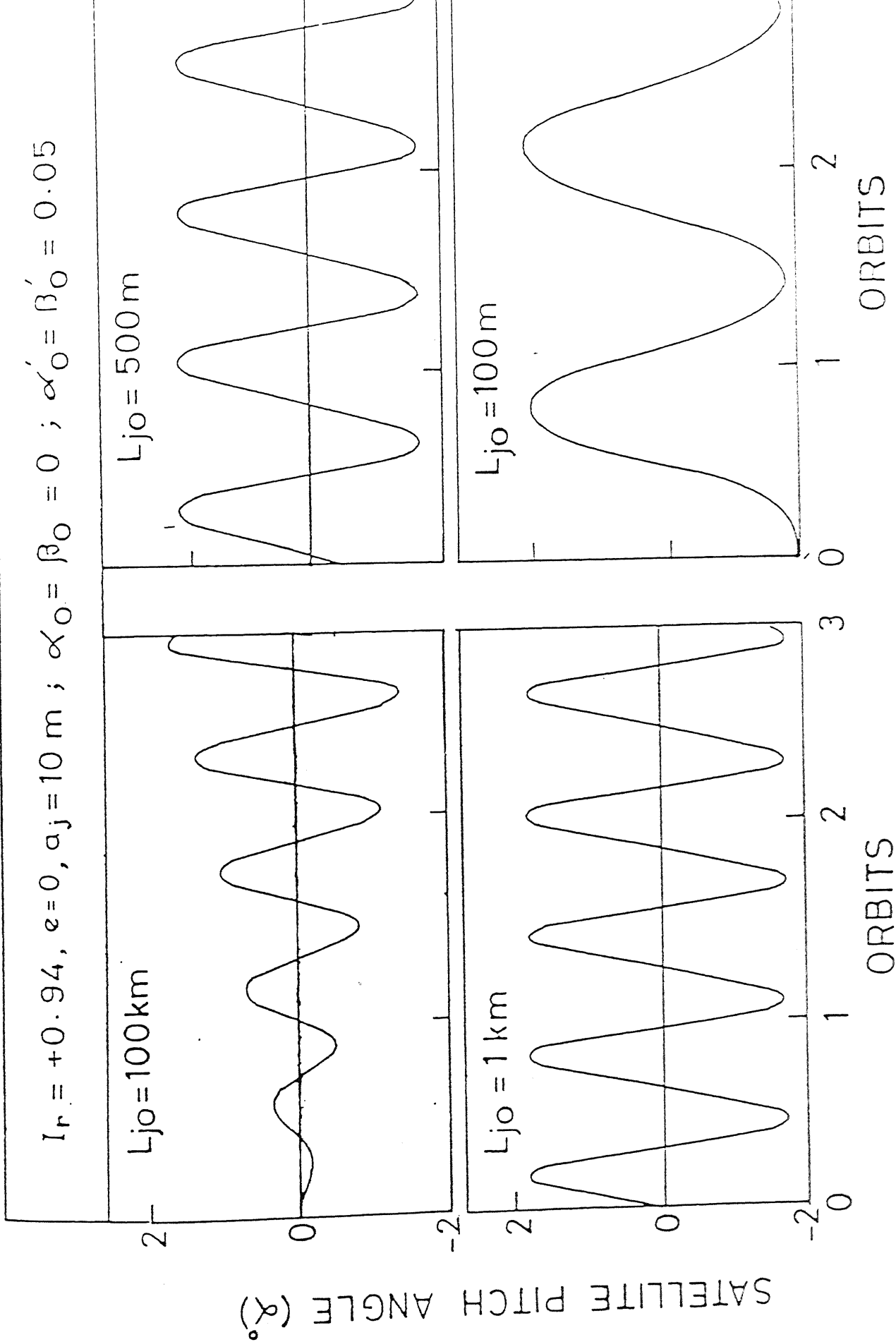


Fig.3. Typical Satellite librational response for various

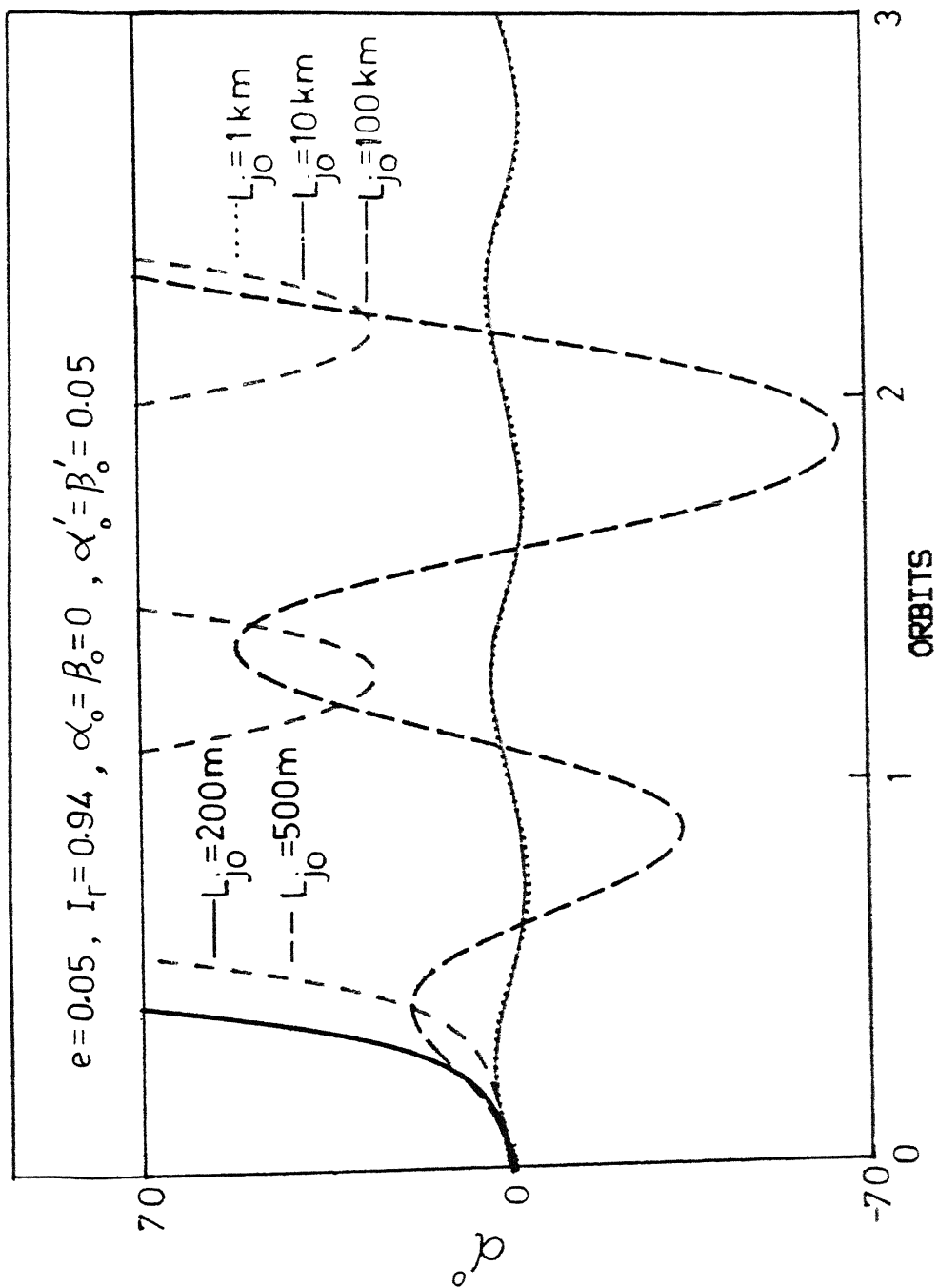


Fig.4. Satellite librational response for various tether lengths at eccentricity,  $e=0.05$

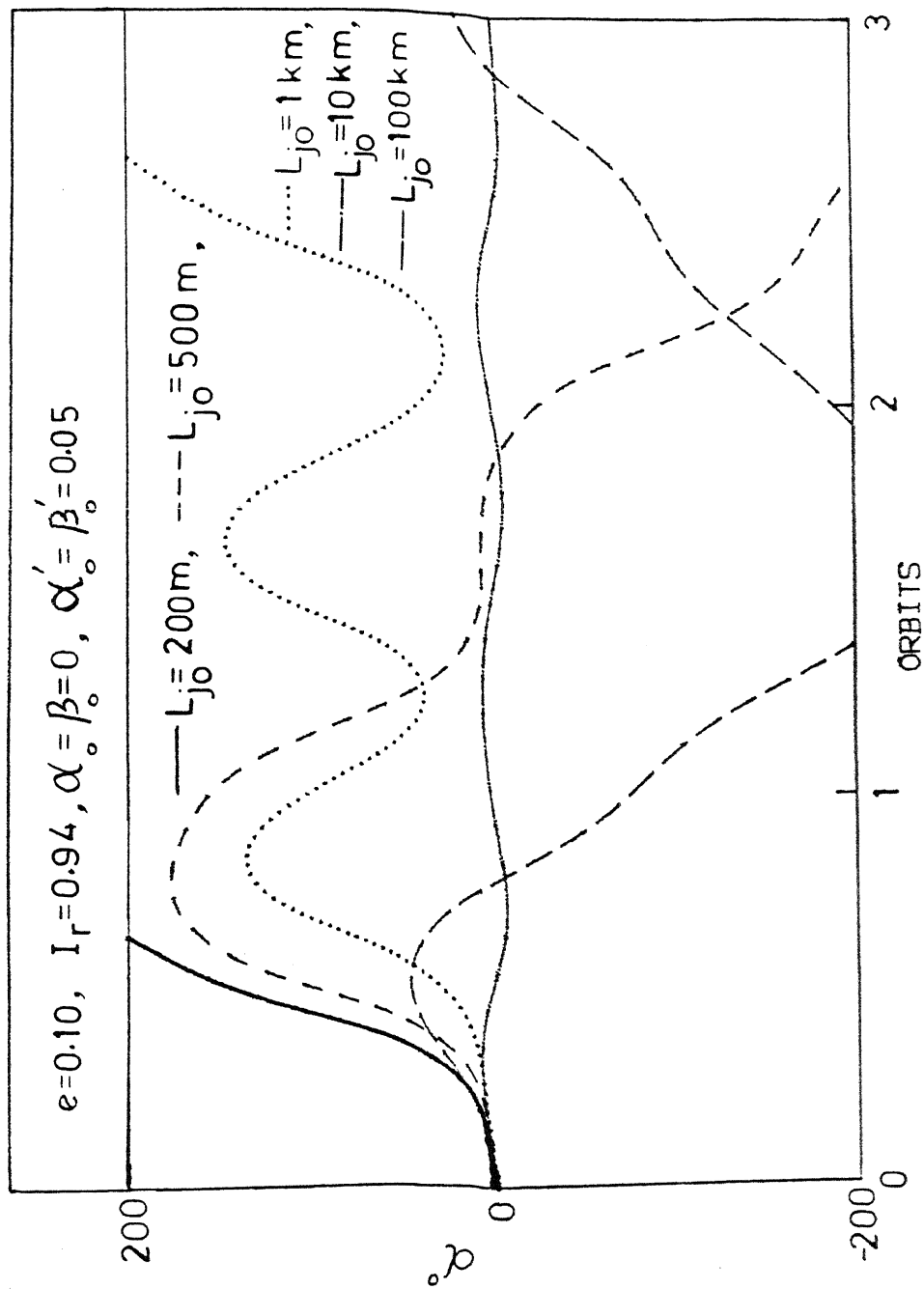


Fig.5. Satellite librational response for various tether

lengths at eccentricity,  $e=0.10$

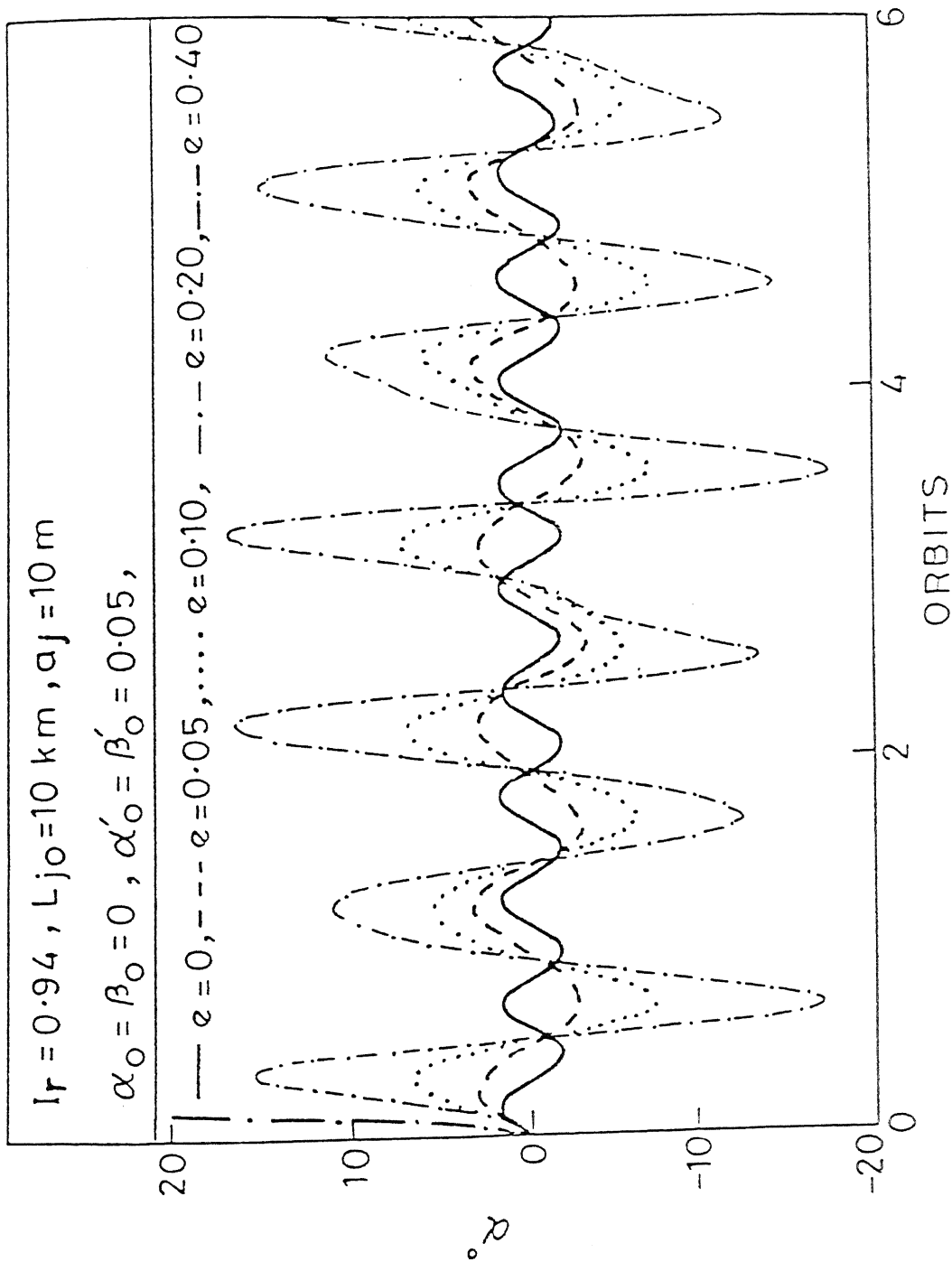


Fig.6. Librational response for Satellites in orbits of

various eccentricities at  $L_{j0} = 10 \text{ km}$

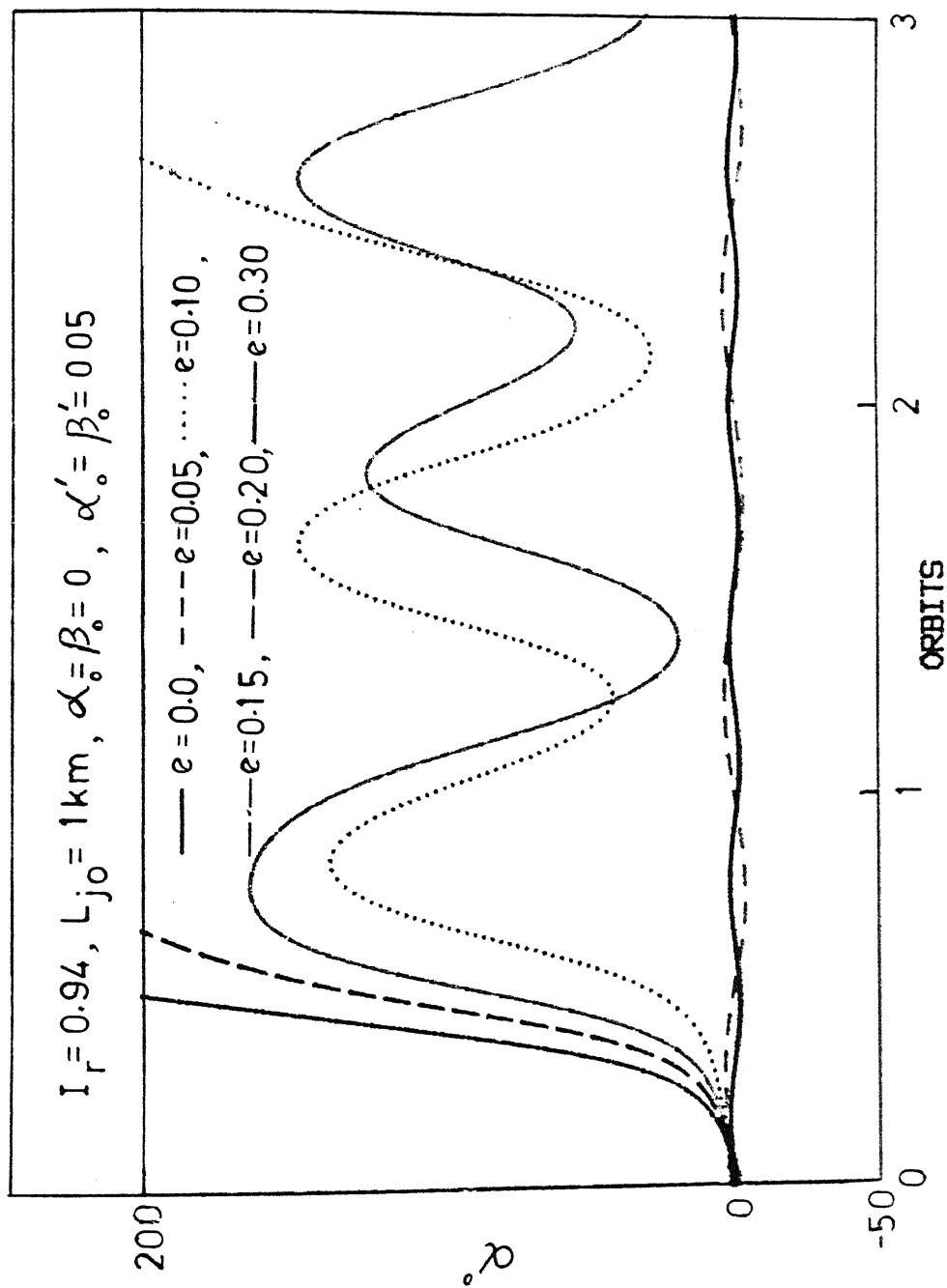


Fig.7. Librational response for Satellites in orbits of

various eccentricities at  $L_{j0} = 1 \text{ km}$

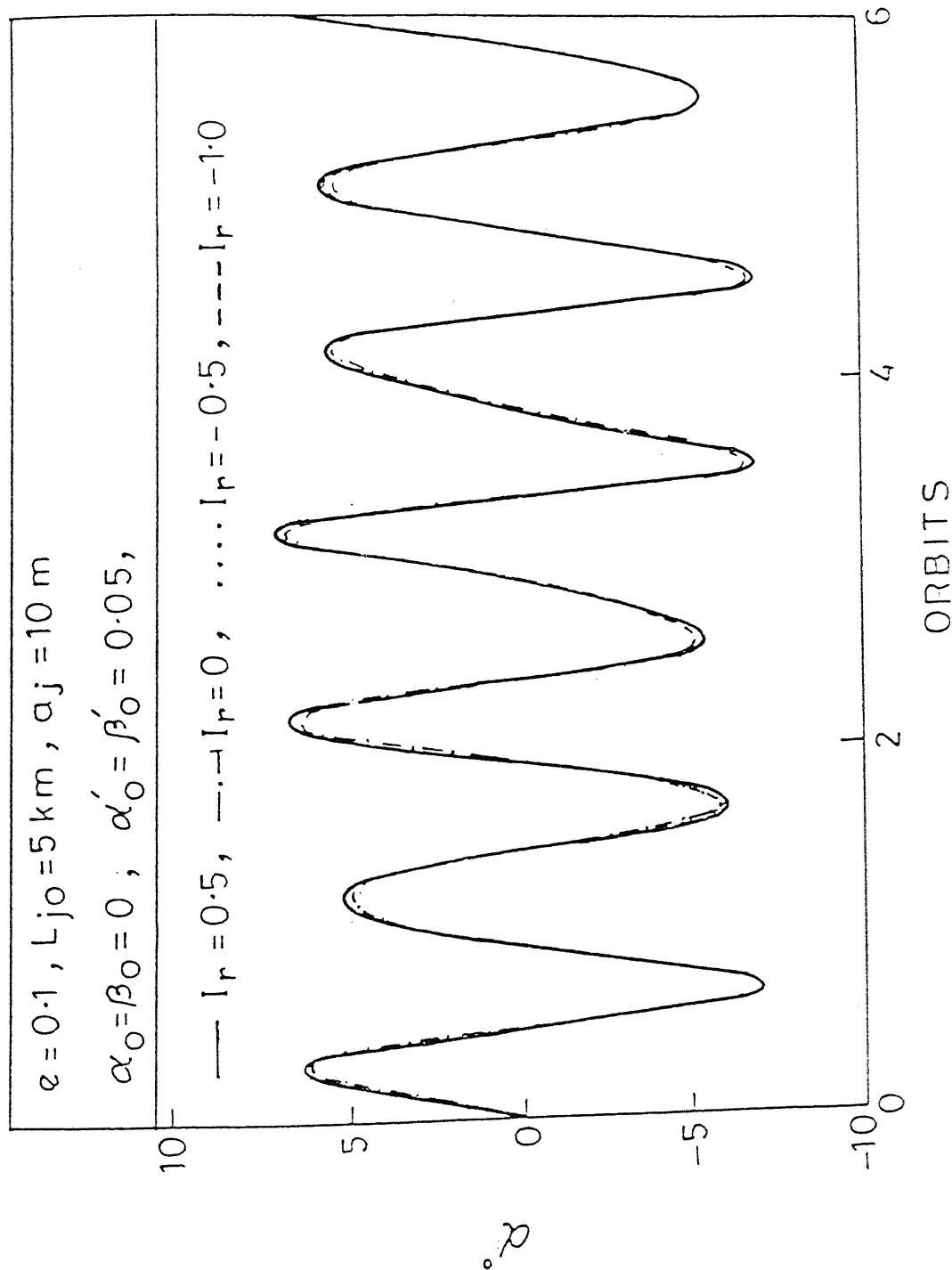


Fig.8. Typical librational response for various Satellite mass distributions at  $e=0.1$

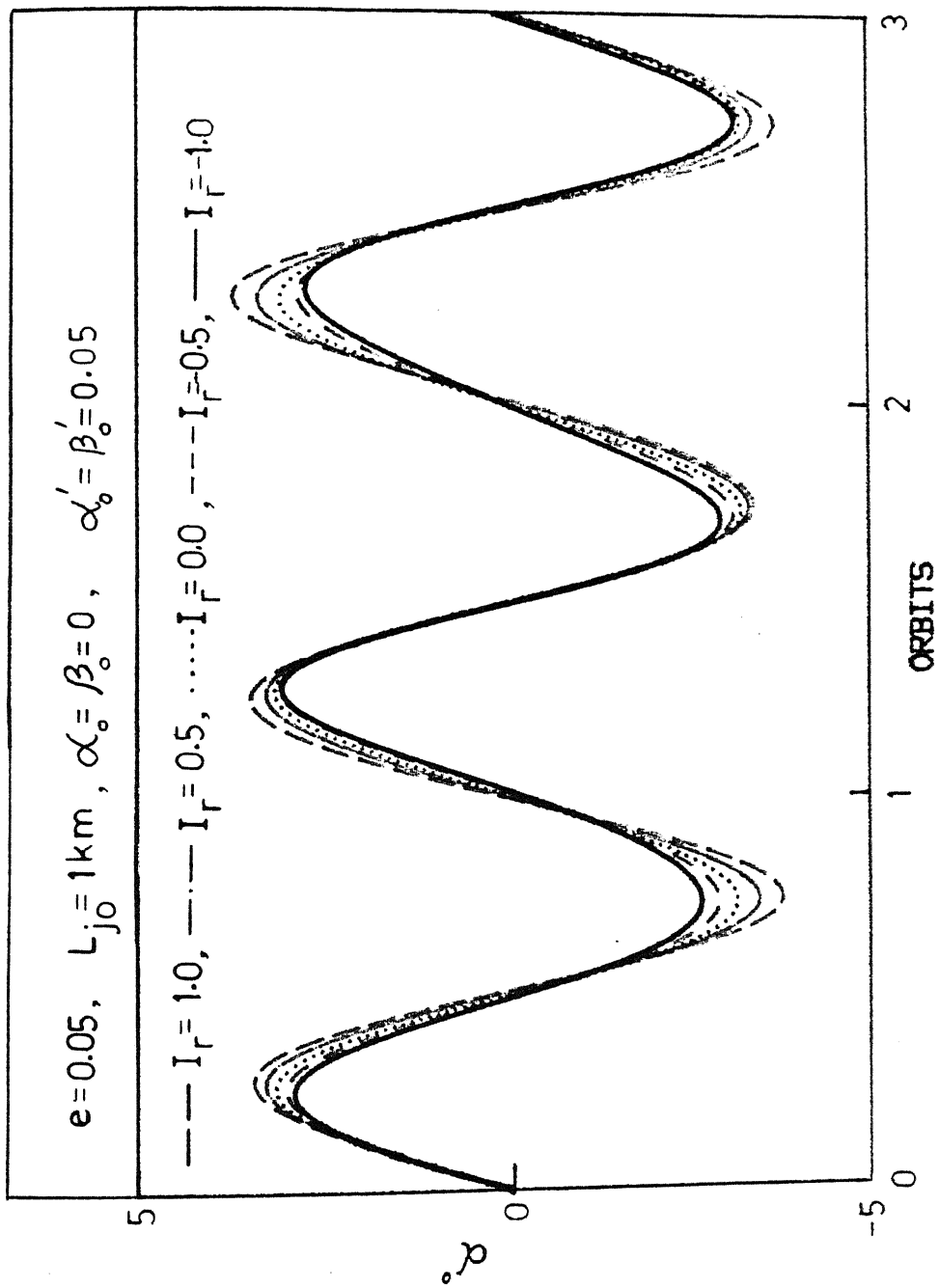


Fig.9. Librational response for various Satellite mass

distributions at  $e=0.05$



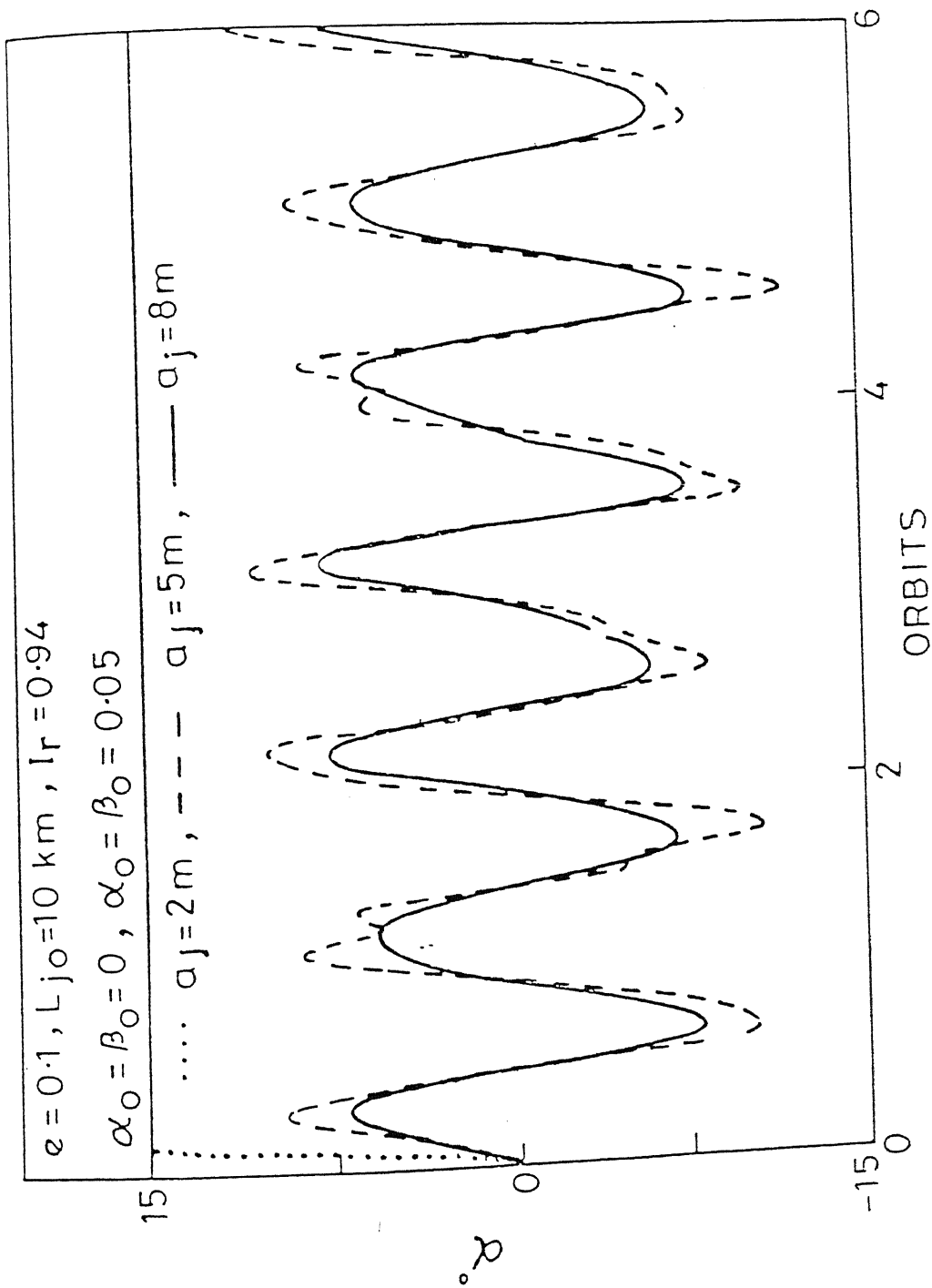


Fig.10. Satellite librational response for various Tether

offsets

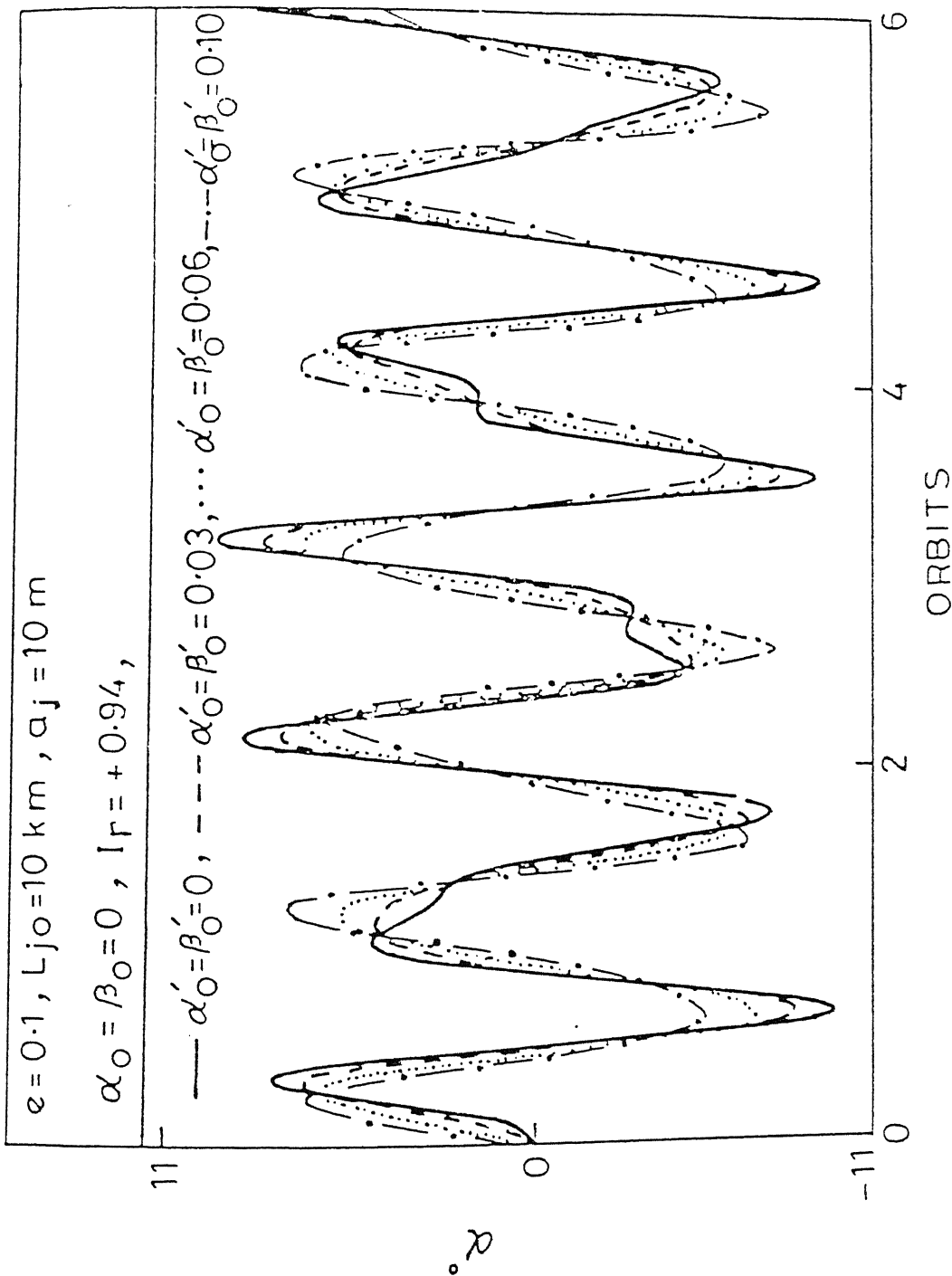


Fig. 11. Satellite librational response for various impulsive

disturbances at  $L_{j0} = 10 \text{ km}$

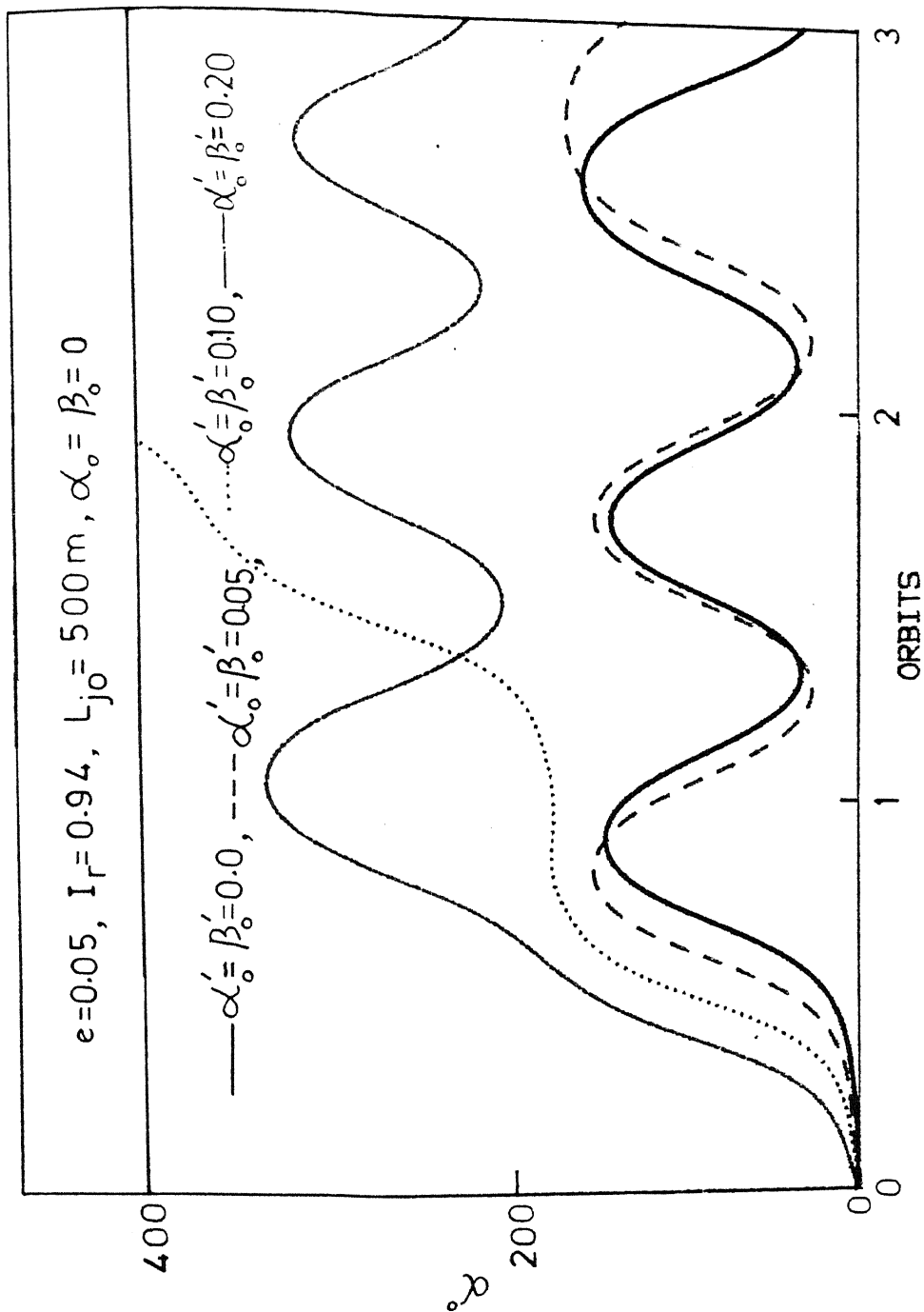


Fig.12. Satellite libration response for various impulsive

disturbances at  $L_{j0} = 0.5 \text{ km}$

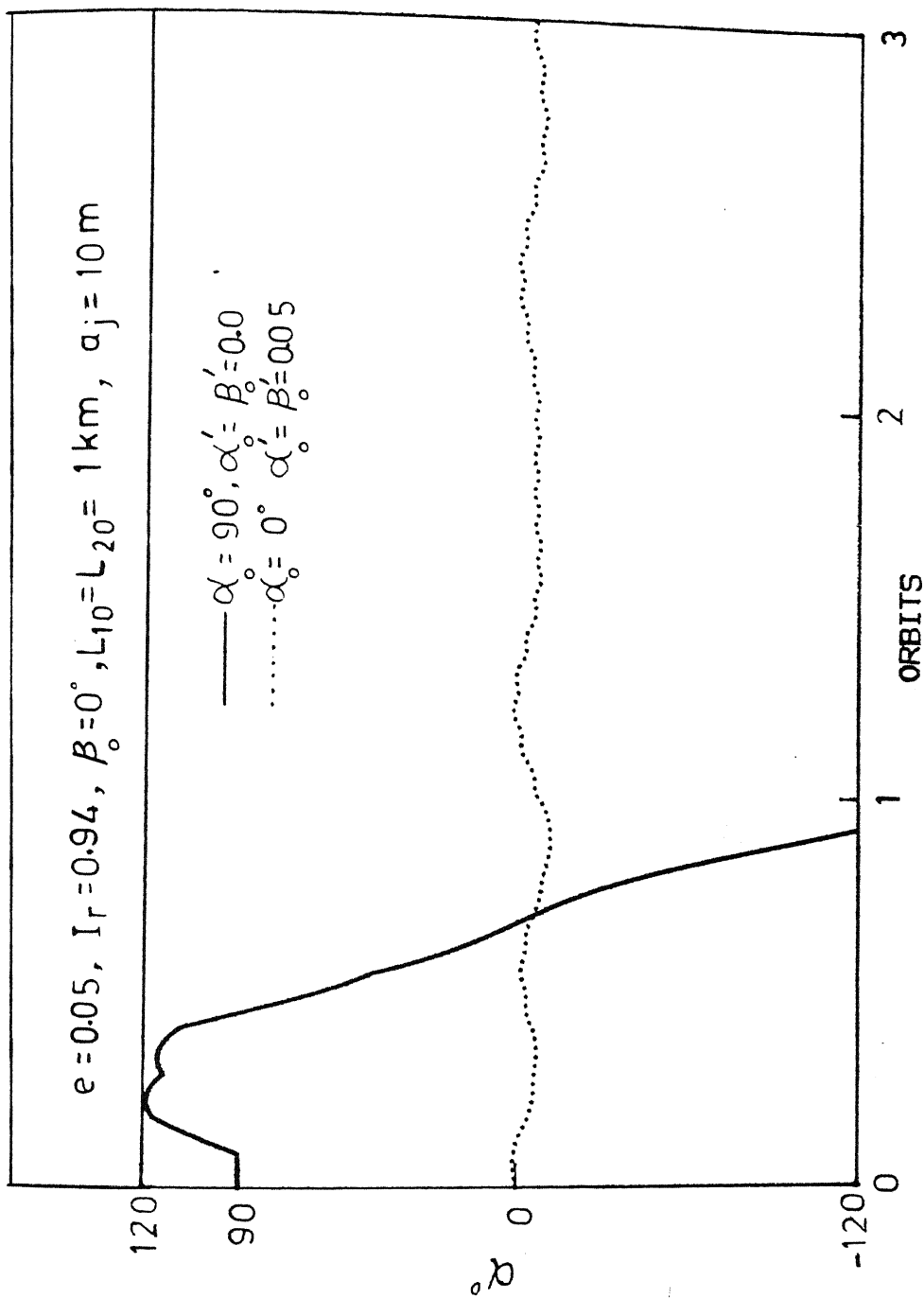


Fig.13. Librational response  $\alpha$  for different configurations and disturbances under deployment

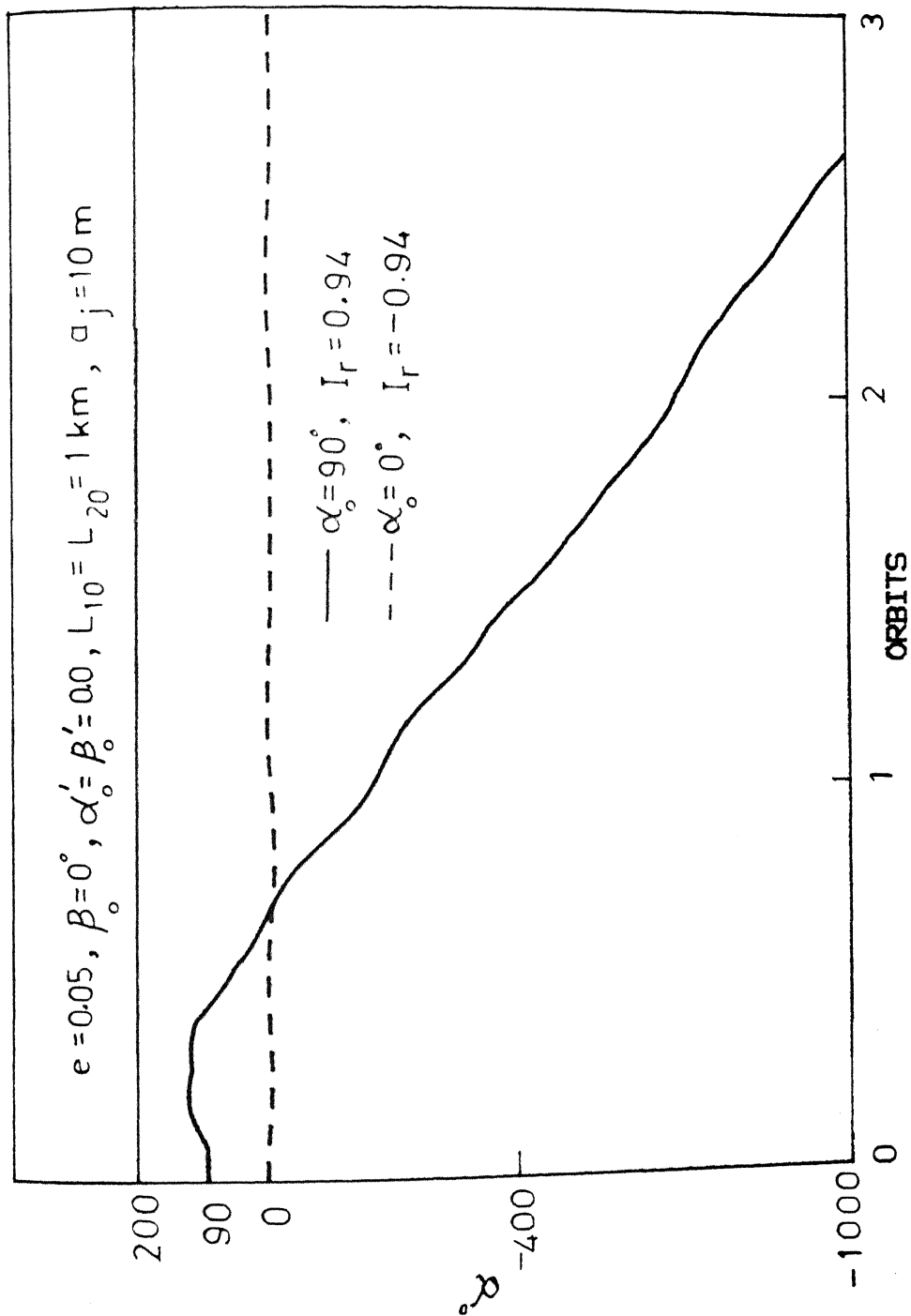


Fig.14. Librational response  $\alpha$  for different configurations and mass distributions under deployment

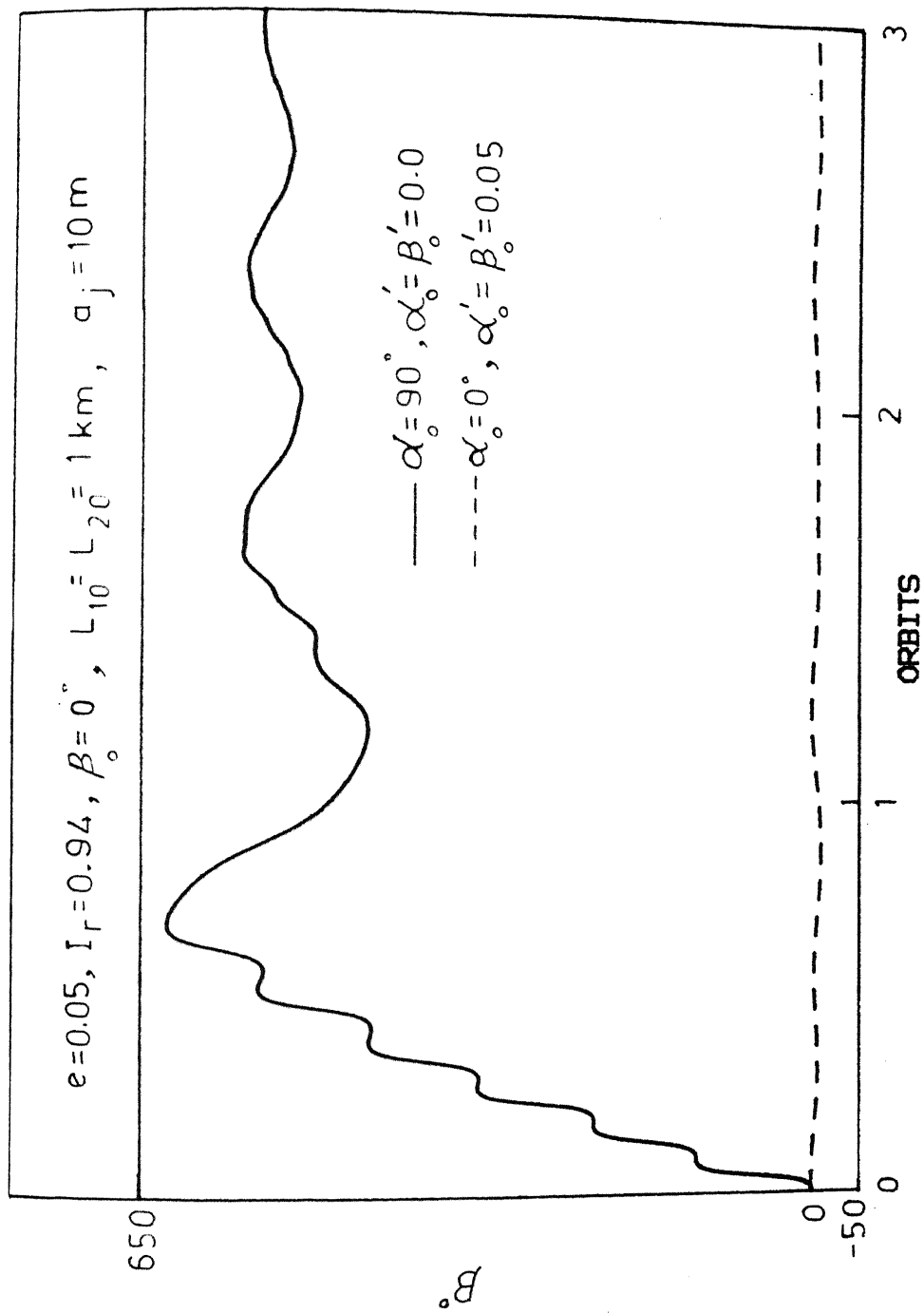


Fig.15. Librational response  $\beta$  for different configurations and disturbances under deployment

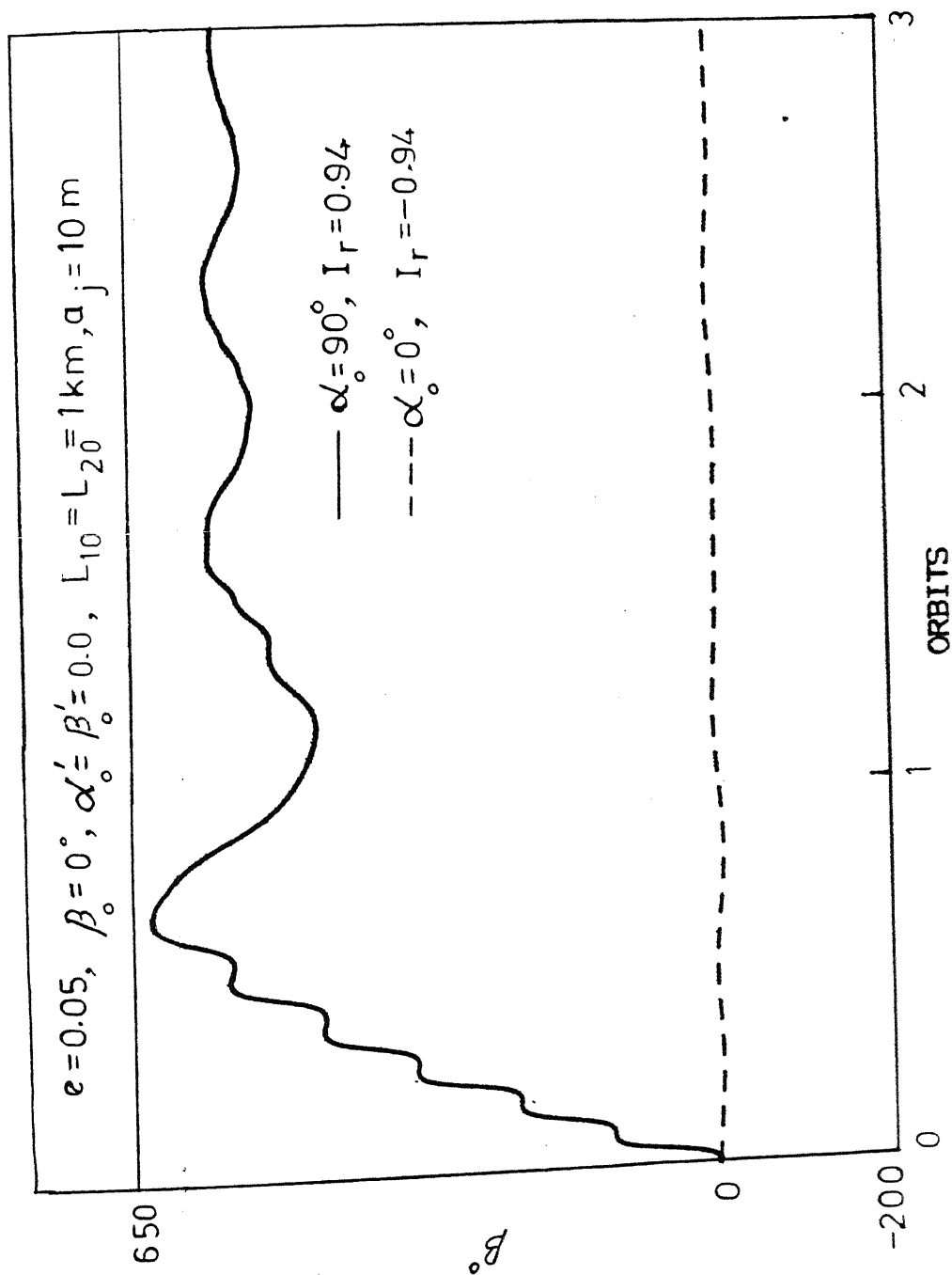


Fig.16. Librational response  $\beta$  for different configurations  
 and mass distributions under deployment

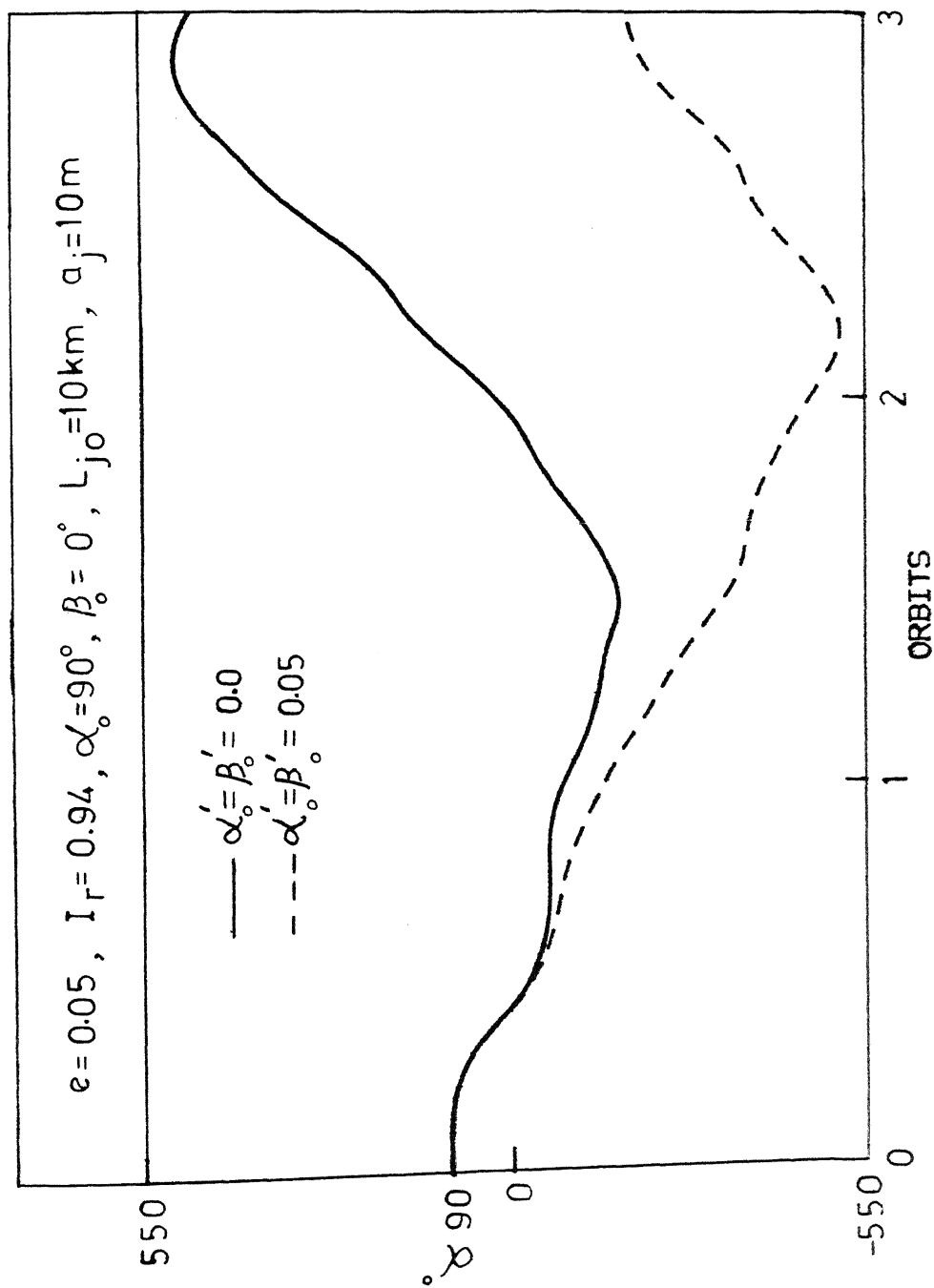


Fig.17. Librational response  $\alpha$  for different disturbances under retrieval



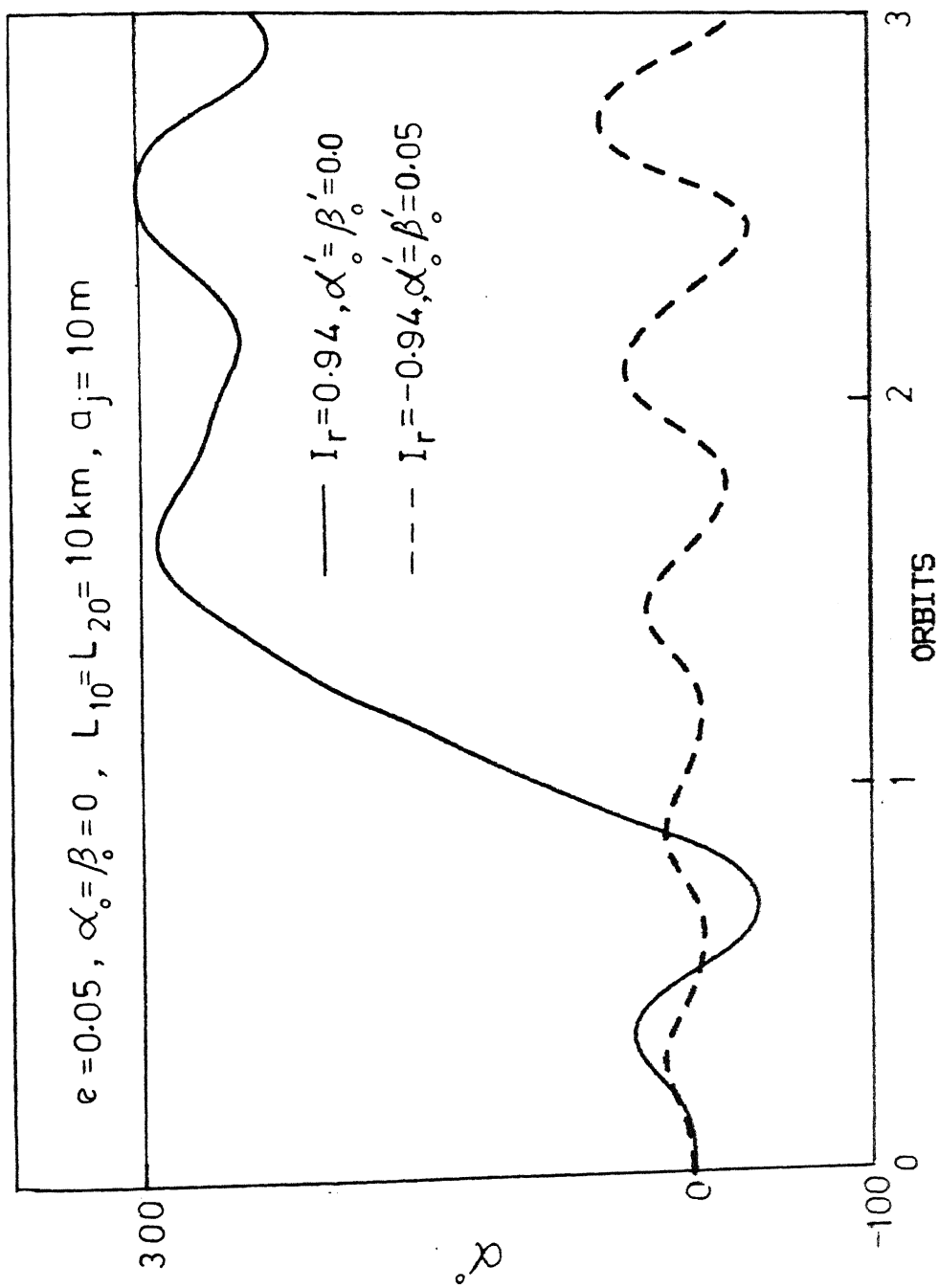


Fig.18. Librational response  $\alpha$  for various impulsive disturbances and mass distributions under retrieval

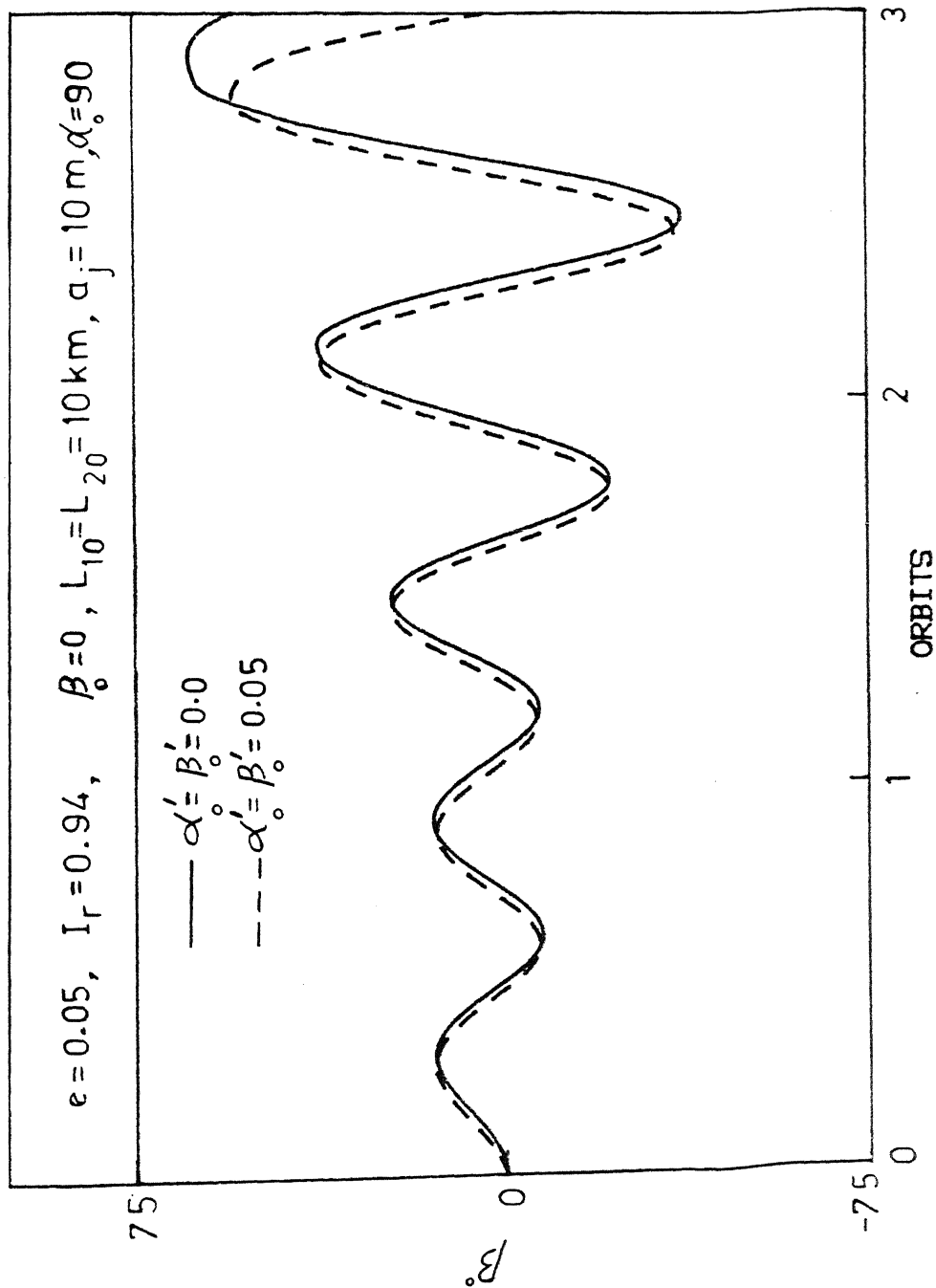


Fig.19. Librational response  $\beta$  for different disturbances  
 under retrieval

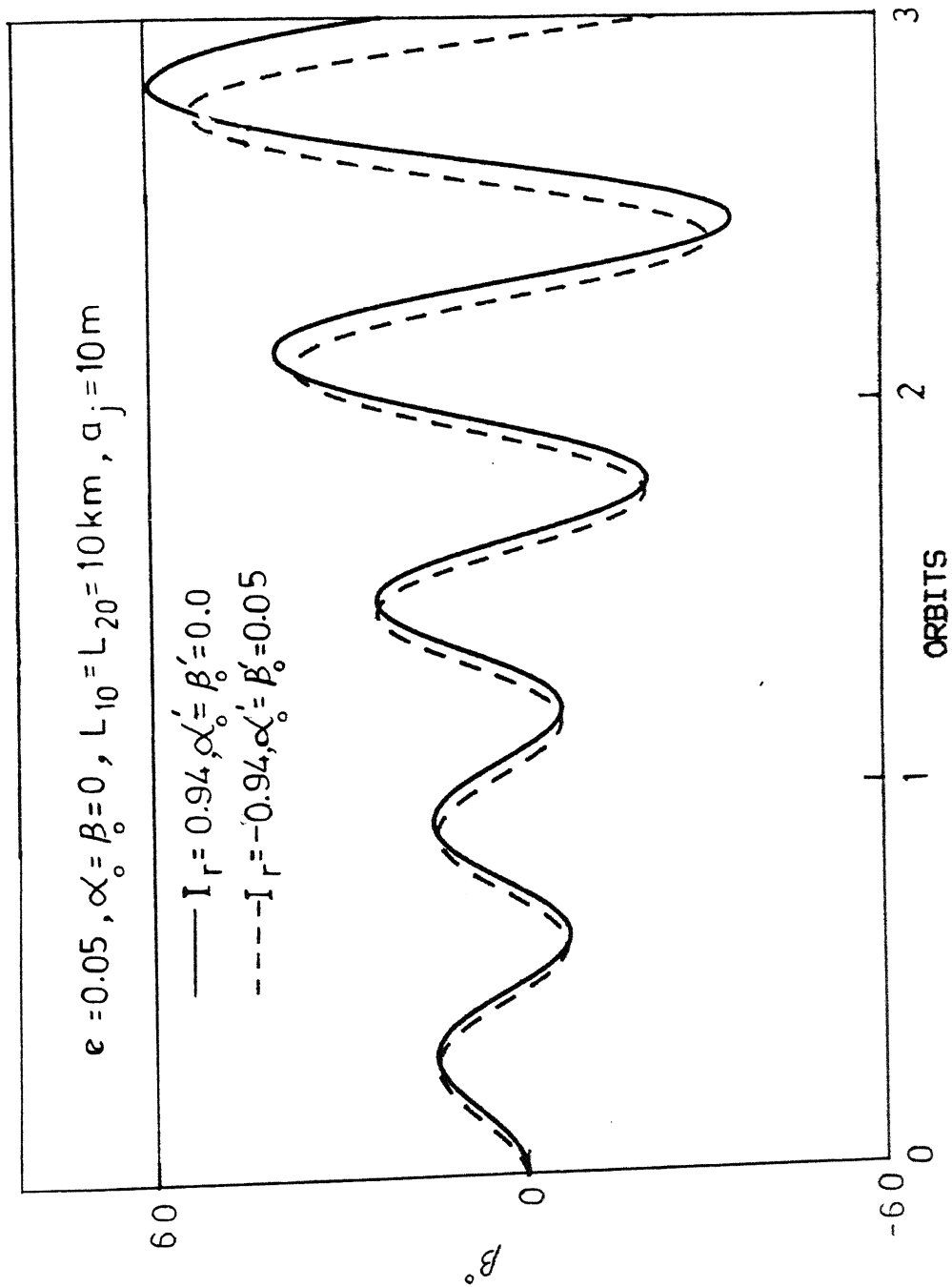


Fig.20. Librational response  $\beta$  for various impulsive

disturbances and mass distributions under retrieval

Publication based on the thesis work :

Kumar, K., Khosla, A., and Chaudhary, K., " Tether as a Satellite Attitude Stabilizer in Elliptic Orbits : A Novel Concept ", Paper No. AAS 93-735, presented in the AAS/AIAA Astrodynamics Specialist Conference, Victoria, B.C., Canada, August 16-19, 1993, to appear in Advances in Astronautical Sciences.